Rise of the Service Sector and Female Market Work: Europe vs US*

Michelle Rendall†
The University of Zurich

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ABSTRACT

Continental Europe has seen a smaller rise in formal female employment compared with the United States or the Nordic countries. Additionally, Continental Europe has a substantially smaller service sector. These facts coincide with job requirements shifting from physical strength to intellectual capacity. Given empirical evidence, this paper develops a model of endogenous technical change, where new “technologies” can be invented to increase the productivity of brain-inputs. Two inputs, brain and brawn, are combined through CES production functions into services and industrial goods, with the production sector for goods requiring more brawn than brain. Households allocate time to working at home or the labor market, choose consumption of services and goods, and invest in new technologies. The key is households can produce a substitute for market services and women have, on average, less brawn than men, giving them a comparative advantage with respect to staying home and working in the service sector. Therefore, an economy that does not facilitate the movement of women into the labor market, by imposing high taxes, causes service production to remain at home. This reduces technological innovation, pushing an economy into a self-reinforcing loop, where a small service sector feeds back into low total hours worked by women (and men), further depressing the service sector.


Keywords: technological progress, sectoral labor allocation, cross-country differences, gender wage gap, labor demand/supply.

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† University of Zurich, IEW, Muehlebachstrasse 86 CH-8008 Zurich. Email: mrendall@iew.uzh.ch. All errors are mine.
1. Introduction

One of the greatest phenomena of the 20th century has been the rise in female labor force participation. However, this phenomena has not been uniform, with Continental Europe experiencing a smaller rise in formal female employment compared with the United States or the Nordic countries. Moreover, Continental Europe has a substantially smaller service sector. The correlation between relative female employment change and aggregate service employment is around 0.82 from the 1980s onward for a large set of OECD countries (Rogerson, 2005). In addition, Rogerson (2007a) finds that when comparing aggregate employment between the United States and Europe, most of this discrepancy is accounted for by differences in service sector employment. While there are small employment differentials in industry (slightly positive), Europe has an employment differential in services of -9.4 percent in 1956 and -15.5 percent in 2000 compared to the United States. The microeconomic literature has found that most labor market differences across countries can be accounted for by difference in time spent working on home production. At the upper bound, Freeman and Schettkat (2002) find total hours worked (home and market) to be very similar for Americans and Germans. Olovsson (2009) finds that in the case of Sweden and the United States, time spent working on home production can account for about 90 percent of the difference in market hours. At the lower end of estimates, Ragan (2006), using the Harmonized Time Use Study, accounts for about one third of the differences in market work.

This study develops a general equilibrium model based on the following three facts of labor supply and sectoral labor allocation across Europe and the United States (see figure 1).

1. Women’s labor force participation in Continental Europe and the Scandinavian countries stayed constant over time compared to the United States.

2. Service sector employment in the two regions remained constant compared to the United States.

3. Differences in total employment have come from a falling goods sector’s (industry and agricultural sector) employment share relative to the United States.
Given data availability, this study defines Continental Europe as France, Germany and Italy, and the Scandinavian countries are defined as Denmark, Finland, Norway and Sweden.

The paper summarizes empirical evidence from the United States for the labor requirements, and their evolution, over time (for further details see Rendall, 2009) across sectors and gender. Given the empirical evidence, a general equilibrium model is developed to understand the evolution of female employment and their rise in the service sector. On the demand side, production is modeled as aggregate constant elasticity of substitution (CES) functions for each sector, industry and services, where brain and brawn are the inputs as in Guvenen and Kuruscu (2009). The economy experiences brain-biased technical change (BBTC) over time, which leads to a rise in the demand for brain inputs. This technical change occurs endogenously with households investing into the development of new brain-augmenting technologies. Similar to Ngai and Pissarides (2008), households invest in new technologies, allocate time between the home and labor market, and choose
consumption over three types of goods: market produced services, market produced goods and home produced goods/services. The model has two key assumptions, similar to Rogerson (2007a), households can produce a substitute for market produced services (e.g., childcare, elderly care, meals) using home production technology and labor time. Second, given United States estimates, it is assumed that the service sector requires less brawn than the industrial sector, and that men and women have the same brain levels, but women have less brawn. Therefore, women generally prefer working in the service economy where occupations do not require much physical strength or have adverse working environments. A rise in the service sector is synonymous with a shift in labor factors away from brawn, as similarly suggested by Galor and Weil (1996), which will lead to an improvement in women’s labor market outcomes, i.e., higher wages.

From the features of sectoral production, home substitution and women’s comparative advantage, a rich set of dynamics capable of generating both a convergence in female and male labor market outcomes, and a rise in the service sector are presented. That is, an economy that does not initially facilitate the movement of women into the labor market, by, for example, imposing high taxes without the social benefits tied to working women in the Nordic countries (e.g., subsidized full-day child care, elderly care) causes the production of services to remain in the home. As a result, women do not enter the workforce and the growth of the service sector outside the home is considerably slowed. This, in turn, provides even fewer incentives for households to invest into brain-complimentary technologies and for women to enter the labor market, since fewer good job opportunities for women are created through lower aggregate demand for services. As a result, the United States and Nordic countries have entered into a self-reinforcing loop, where female employment increases demand for market produced services, thus increasing the demand for female workers and leading to more investment into brain-augmenting/brawn-substituting technologies. However, in the Nordic system, women enter the labor market through a tax-subsidy distortion, leading to welfare losses.

The model is calibrated to match the evolution of female employment in the United States in the 1950s and 2000s. I first determine how much of the rise in female employment can account for the rise in the service sector and how much is due to other technological progress. Using the calibrated
economy of the United States, we can analyze what would have happened if taxes were at the level of Continental Europe. I.e., how much would female employment have grown? How large would the service sector be? Furthermore, in the context of the model, we can also study the differences between the United States (low taxes) and the Nordic countries (high taxes and subsidized market services) with respect to social welfare loses, by setting a subsidy to services which is tied to women’s labor force participation that matches female employment levels in the United States.

While the model abstracts from other factors of the rise in female employment (e.g., improvements in home technology (Greenwood and Guner, 2008, see)) and growth (e.g., capital-skill complementarity, education), the exercise is useful for various reasons. First, while there exits a large literature accounting for the differences in aggregate labor between Europe and the United States over the last 40-50 years, most notably Prescott (2004); Rogerson (2005) who show that taxes can account for a large fraction of differences in changes of hours worked between the two regions, this study points to a mostly ignored fact in the study of labor market differences between Europe and the United States, i.e., female employment. Second, it furthers the discussion on why some countries have not developed a large service sector. Rogerson (2007a) shows that taxes and technology can account for the difference in both hours worked and in sectoral labor allocations, with a substantially lower service sector in Continental Europe. Rogerson (2007b) also explains the large service sector in the Nordic countries. Albeit high tax rates substantial subsidies to services, such as childcare, elderly care, etc, can lead to a large service sector with low total hours worked. Lastly, this study highlights the differences that have pushed women into the labor market, contrasting the United States and the Nordic countries and their taxation policies’ associated welfare loses. Most of the literature that seeks to account for the rise in female labor force participation has focused on demand driven stories only, i.e. improvements in home technology, such as the invention and marketization of household appliances (see, for example, Greenwood et al., 2002, and references therein), or the improvements in baby formulas (see Albanesi and Olivetti, 2006), rising female wages (see Jones et al., 2003) and returns to experience (see Olivetti, 2006), or the effects of cultural, social, and intergenerational learning on labor supply (see Fernández, 2007; Fogli and Veldkamp, 2007). Here, the rise in female labor force participation is instead the result of a change in the demand of labor requirements, in the
spirit of Galor and Weil (1996) or Guvenen and Kuruscu (2009). This evidence is central for any country that is pursuing increased economic growth by incentivizing women to enter the labor market.

As labor demand changes are the key motivation for this study, Section 2 provides further evidence for the changing labor market, focusing on the evolution of physical and intellectual job requirements in the United States and women’s self-selection into low-strength jobs. The general equilibrium model is outlined in Section 3, and Section 4 provides analytical results of BBTC on labor demand, labor supply, wages, and sectoral labor shares. Section 5 discusses the estimation and calibration procedure, and Section 6 presents labor market trends across regions resulting from differences in taxation and work subsidies. Lastly, Section 7 concludes.

2. United States Labor Facts

To explore the relationship between the rise in the service sector, female labor force participation and changes in labor demand, this study focuses on the relative demand and supply of two types of labor inputs: intellect and physical strength. This study starts from the premise that women have, on average, less brawn than men. Accepting that women and men have similar levels of brain, men have a comparative advantage in brawn-intensive occupations. However, technological change shifts labor demand toward low-brawn occupations, diminishing men’s comparative advantage in the labor market.

Using factor analysis (FA), in a related paper (see for details Rendall, 2009), I obtain brain and brawn estimates by United States census occupation and industry classifications from the 1977 Dictionary of Occupational Title (DOT). The 1977 DOT reports 38 job characteristics for over 12,000 occupations, these characteristics capture the heterogeneity across jobs and workers. While these characteristics measure different specific job requirements, they can be grouped into broader categories of skills in terms of their common underlying dimensions. This grouping reduces the dimensionality of heterogeneity allowing factor requirements to be matched in a simple general equilibrium model. Characteristics in the DOT document the (1) general educational development, (2) specific vocational training, (3) aptitudes required of a worker, (4) temperaments or adaptability requirements, (5) physical
strength requirements, and (6) environmental conditions required to preform at an average level for a given job. For example, general educational development measures the formal and informal educational attainment required to preform a job effectively by rating reasoning, language and mathematical development. Each reported level is primarily based on curricula taught in the United States, where the highest mathematical level is advanced calculus, and the lowest level only requires basic operations, such as adding and subtracting two-digit numbers. Specific vocational preparation is measured in the number of years a typical employee requires to learn the job tasks essential to perform at an average level. Eleven aptitudes required of a worker (e.g., general intelligence, motor coordination, numerical ability) are rated on a five point scale, with the first level being the top ten percent of the population and the fifth level compromising the bottom ten percent of the population. Ten temperaments required of a worker are reported in the 1977 DOT, where the temperament type is reported without any numerical rating. An example of a temperament is the ability to influence people in their opinions or judgments. Physical requirements include a measure of strength required on the job, rated on a five point scale from sedentary to very heavy, and the presence or absence of tasks such as climbing, reaching, or kneeling. Lastly, environmental conditions measure occupational exposure (presence or absence) to environmental conditions, such as extreme heat, cold, and noise. Similarly to Ingram and Neumann (2006) to reduce the dimensionality of DOT job characteristics one can use factor or principal component analysis. Using FA, a linear relationship between normally distributed broad skill categories (e.g., brain, brawn, motor coordination) and the 38 DOT characteristics is estimated from the associated 38 variable correlation matrix. For a detailed explanation of the estimation procedure see Appendix A of Rendall (2009). The 1977 DOT was subsequently updated in 1991, adding further characteristics for a total of 53\(^1\). Repeating FA with this updated data set results in similar results over time. However, a direct comparison of trends shifting towards brain from 1977 to 1991 between the two datasets with FA is not possible, given the nature of FA where factors are computed with zero mean and standard deviation of one.

\(^1\)Most of the characteristics continued to record the same information, but some provide more details, e.g., the exposure to environmental conditions and physical tasks were recorded by relative usage (frequently, occasionally, etc.) rather than a simple presence test in 1977 (required or not).
2.1. Brain and Brawn in the United States

Using maximum likelihood estimation methods, three factors, brain, brawn and motor coordination, are determined sufficient in capturing the information contained in the 1977 and 1991 DOT characteristics. This study focuses on the first two factors, brain and brawn. Table 1 shows a selection of characteristics that are important (have a large factor loading) on the two factors respectively.

<table>
<thead>
<tr>
<th>Brain Factor</th>
<th>Brawn Factor</th>
</tr>
</thead>
<tbody>
<tr>
<td>Reasoning, Mathematical Development</td>
<td>Repetitive Work</td>
</tr>
<tr>
<td>Language Development</td>
<td>Climbing/Balancing</td>
</tr>
<tr>
<td>Dealing with People</td>
<td>Stooping/Kneeling/Crouching/Crawling</td>
</tr>
<tr>
<td>General Intelligence</td>
<td>Strength Requirement</td>
</tr>
<tr>
<td>Verbal, Numerical Aptitude</td>
<td>Environmental Exposure²</td>
</tr>
<tr>
<td>Directing/Controlling</td>
<td>Indoor or Outdoor Work</td>
</tr>
</tbody>
</table>

These factors are merged with the 1940-1960 United States Census data and the 1968 to 2005 Current Population Survey (CPS) data to compute trends over time.³ Figure 3, which plots all 1977 occupational brain and brawn combinations, clearly depicts the difference in brain and brawn requirements across the economy.

To compute aggregate factor demand changes in the United States over time, occupation-industry factor⁴ estimates are aggregated using United States Census and CPS civilian labor force weights. Figure 2 depicts factor standard deviations from the mean by sector over time, with a normalized mean of zero in 1970. While both sectors have gained in occupations with brain requirements, the service sector has always had a low level of brawn requirement.

³Census and CPS data is obtained from the IPUMS-USA (Ruggles et al., 2004) and the IPUMS-CPS project (King et al., 2004). The IPUMS projects provide a consistent 1950 United States Census classification of occupations and industries over the years, which is used in merging 1977 DOT and 1991 DOT brain and brawn factors.
⁴For the 1991 DOT only a merge on occupations, not industries is possible.
Overall the brain supply steadily increases and the brawn supply steadily decreases. This rising trend in the supply of brain versus the falling trend in the supply of brawn is termed BBTC in this study. These trends are not specific to the 1977 or 1991 DOT alone.\textsuperscript{5} Note that using a single DOT survey to determine job requirements implies that the specific job factor requirements did not change over the last five decades. For example, a craftsman utilized the same brawn level in 1950 as in 2005. Ergo, all trends pictured are due to changes in the composition (mix) of occupations within the economy, and the rise in brain and fall in brawn requirements might possibly be greater than shown due to intra occupation skill-biased technical changes.

Figure 3 depicts brain and brawn standard deviations by gender over time, with the selection of women into occupations with low brawn requirements clearly evident. Given women’s lower innate brawn levels, this bias toward low brawn occupations can be either due to employee self-selection

\textsuperscript{5}Ingram and Neumann (2006) obtain similar trends over time using the 1991 DOT (see Figure 3 in the referenced paper).
or employer discrimination. Goldin (1990) observes that as far back as the 1920-1930s women made work choices based on the level of brawn required. 

Clerical work was cleaner and less strenuous than manufacturing work ... It is understandable why young women preferred office work and why the growth of the clerical sector would lead to the continued employment of women after marriage and childbearing. ... If the considerable difference in the earnings of males and females in manufacturing was largely due to rewards to strength, then the replacement of brain for brawn work should have evened starting salaries. ... Although the difference in starting salaries implied by the earnings functions between unmarried male and female clerical workers was negligible, it was 47% in manufacturing. Extract from Goldin, Understanding the Gender Gap (1992) pp. 108-109

This evidence gives strong support of the hypothesis for productivity differences across sector employment rather than overall labor market discrimination.
Given the above facts, a general equilibrium model of BBTC, concentrating on the service sector and female employment is presented in the following section.

3. General Equilibrium Model

The simulated economy consists of a representative of household, composed of a man and a woman and two competitive production sectors, industry and services, and a government.

3.1. Technology and Preferences

Government

The government, who solves a balanced budget, taxes individuals labor income at rate, \( \tau \) (yielding the underlying cross-country differences over time). Tax revenues are rebated to households as a lump-sum transfer, \( T \), price subsidizes on market services, \( v \), and/or rebates for service goods indexed to women’s labor supply as in Ragan (2006), \( \phi \).

\[
\tau (w^m_t h^m_t + w^f_t h^f_t) = T_t + v p_t c_{st} + \phi h^f_t, \tag{1}
\]

where \( w^m_t, w^f_t \) are male and female wage detailed below and \( h^m_t, h^f_t \) are the respective labor supplies.

Production

The competitive sectors combine the two aggregate inputs, brain and brawn, \( \{B, R\} \), supplied by households, in a CES production function to produce final goods/services, \( \{Y_i, Y_s\} \). By assumption the production of industrial goods is more efficient in using brawn inputs (requires more brawn) than the service sector. There are two types of final sectors, the industrial and service sector. Each production process uses all inputs, brain \( B_b \equiv B \) or brawn \( R_r \equiv R \), where \( B \) and \( R \) are the aggregated individual labor supplies.

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6While the rise in labor force participation was considerably greater for married women, the current model focuses on only married households. Adding single households does not provided further dynamics to the model, not affecting any of the qualitative results.
of brain and brawn. These brain and brawn units are combined in a CES production function to produce the final market good,

\[ Y_{jt} = Z_t \left[ \alpha_j (M_{bt} B_{jt})^\rho + (1 - \alpha_j) (M_{rt} R_{jt})^\rho \right]^{1/\rho} \text{ for } j = i, s, \tag{2} \]

where \( A_{bt} \) factor productivity of brain inputs; \( \epsilon_{\phi} = 1 / (1 - \phi) \) is the elasticity of substitution between the two inputs; \( \alpha_j \) is occupation \( j \)'s production share in brain, \( Z_t \) is total factor productivity and \( B_{jt}, R_{jt} \) are aggregate factor supplies for each sector \( j \). By assumption of brawn-intensity of the sectors, \( \alpha_s > \alpha_i \).

The study will focus on relative BBTC, that is \( M_{rt} \equiv 1 \), consequently, a change in \( M_{bt} \) over time represents the technical progress. Therefore, firms maximize by choosing brain and brawn factor demands,

\[ \max_{B_{jt}^d, R_{jt}^d} p_{jt} Y_{jt} - w_{jbt} B_{jt}^d - w_{jbt} P_{jt} \tag{3} \]

Household Preferences

Household members are indexed with the superscripts \( g \in \{ f, m \} \) for their gender. The only difference between genders is the lower innate brawn levels of women. Without loss of generality, innate intellectual abilities, \( b^m = b^f \equiv 1 \), and innate brawn \( r^m > r^f \equiv (1 - \tau_r) \), are normalized. There is no bargaining in the households and the households solves a unitary utility \( u(C, L) \), by allocating the labor time of both agents to the market, home production, and leisure; purchasing industry goods, \( c_i \), and services, \( c_s \), in the market, and producing home produced service substitutes at home, \( c_n \).

\[ \max_{\{c_i, c_s, h_t^m, h_t^f, n_t, m_{bt+1}\}} \sum_{t=0}^{\infty} \beta^t a_c \log(C_t) + (1 - a_c) \log(L_t) \tag{4} \]

s.t.

\[ p_i c_{it} + (1 - v)p_sc_{st} + p_t m_{bt+1} = (1 - \tau)(w_i^m h_t^m + w_i^f h_t^f) + T_t \text{ and } \tag{5} \]

\[ 1 = h_t^m + n_t^m + \ell_t^m, \ 1 = h_t^f + n_t^f + \ell_t^f, \ n_t = n_t^m + n_t^f, \tag{6} \]

where \( C \) is the consumption composite of services and goods (suppressing time subscripts), i.e.,

\[ C = (a_i c_i^t + (1 - a_i) F(\hat{c}_s, c_n^t))^{1/\tilde{\tau}}, \tag{7} \]
where $F(\hat{c}_s, c_n)$ is the service composite, i.e.,

$$F(\hat{c}_s, c_n) = \left( a_c \left( c_s + \phi h^f \right) \right)^\eta + \left( 1 - a_c \right) c_n^\eta,$$

(8)

where $\hat{c}_s = c_s + \phi h^f$ are the total market purchased services, both privately and rebated by the government for female hours worked. Home production is linear in labor $c_n = M_n n$. The choice variable $m_{bt1}$ is innovation into new brain-augmenting technologies is described in detail below. Lastly, leisure is assumed to be Leontief, i.e., husband and wife prefer spending time together when not engaged in work $L = \min\{1 - h^m - n^m, 1 - h^f - n^f\}$, which implies that leisure $\ell^m = \ell^f \equiv \ell$.

**Wages**

Since individuals supply both brain and brawn in fixed quantities when they work, and sectors use these inputs in different quantities, there are four wage rates, $\{w_{ibt}, w_{irt}, w_{sbt}, w_{srt}\}$. Thus, an individual that works $h_{jt}^g$ fraction of time in sector $j$ will earn a wage $w_{jt}^g = w_{ibt} b^g + w_{irt} r^g$. Note, due to free mobility of labor, individuals will allocate to the sector where they can earn higher wages. Therefore, if man works in both sectors, his wage is $w^m_t = w_{ibt} + w_{irt} = w_{sbt} + w_{srt}$, given assumptions on brain and brawn above. As a consequence, with the assumption on $\alpha_j$ and $r^f$, women will work only in the service sector and earn a wage $w^f_t = w_{sbt} + w_{srt} (1 - \tau_r)$. Similarly, if women work in both sectors, their wage is $w^f_t = w_{ibt} + w_{irt} (1 - \tau_r) = w_{sbt} + w_{srt} (1 - \tau_r)$ and male wages are $w^m_t = w_{ibt} + w_{irt}$. Therefore, men and women's wages will always be the following,

$$w^m_t = w_{ibt} + w_{irt}, \quad \text{and} \quad w^f_t = w_{sbt} + w_{srt} (1 - \tau_r).$$

(9)

(10)

In addition, total factor supply by sectors is then, if men work in both sectors,

$$B^s_{it} = h^m_{it},$$

(11)

$$R^s_{it} = h^m_{it},$$

(12)

$$B^s_{st} = h^m_{it} - h^m_{it} + h^f_{it}, \quad \text{and}$$

(13)

$$R^s_{st} = h^m_{it} - h^m_{it} + h^f_{it} (1 - \tau_r).$$

(14)
or

\begin{align}
B_{st}^s &= h_t^m + h_t^f (1 - \tau_r), \\
R_{st}^s &= h_t^m h_t^f (1 - \tau_r), \\
B_{st} &= h_t^f - h_t^f, \text{ and } \\
R_{st}^s &= (h_t^f - h_t^f) (1 - \tau_r) 
\end{align}

Invention of New Technologies

Households can invest in the invention of new technologies.\(^7\) New technologies are produced through the use of machinery and the technology stock available. Households purchases machines, \(x_{bt}\), produced by the industrial sector at the price \(p_{it}\) towards the invention of new technology. New technologies are produced by

\[ M_{bt+1} - M_{bt} = \delta M_{bt}^{\psi_m} x_{bt}^{\psi_x}, \quad (19) \]

where \(\psi_m < 0\) implies increasing costs of producing new technologies with the amount of available machines, leading to innovation without population growth similar to Jones (2005) will eventually stop. It follows from (19) that in order for the brain-augmenting factor to grow by \(M_{bt+1} - M_{bt}\), households need to invest

\[ m_{bt+1} = \left( \frac{M_{bt+1} - M_{bt}}{\delta M_{bt}^{\psi_m}} \right) \frac{1}{\psi_x}, \quad (20) \]

in terms of machines purchased at price \(p_{it}\). Note that productivity differences for women give households an incentive to invest in brain-augmenting technologies, as long as woman do not spend too much time on home production.

Decentralized Equilibrium

An equilibrium, given wages \(\{w_{ibt}, w_{irt}, w_{sbt}, w_{srt}\}\), market prices \(\{p_{it}, p_{st}\}\), and government prices \(\{\tau, v, \phi\}\), consists of the time path of households’

\(^7\)It is possible to add another layer equivalent to the endogenous growth literature following Romer (1990) by allowing monopolists to invent new patents for intermediate goods. As this study is not in particular concerned with the balanced growth path, but is rather focused in explaining female employment and sectoral labor shares along a transition, we omit this layer (which was included in a previous version, but did not significantly alter the results, since households were by assumption the owners of the intermediate monopolists).
allocation \{c_{it}, c_{st}, m_{bt+1}, h_t^m, h_t^f, n_t\}, firm factor demands \{B_{it}^d, R_{it}^d, B_{st}^d, R_{st}^d\}, output \{Y_{it}, Y_{st}\} and government allocation \{T_t\} such that for all \(t\):

1. \(\{c_{it}, c_{st}, m_{bt+1}, h_t^m, h_t^f, n_t\}\) solves the Household Problem (4);
2. \(\{B_{it}^d, R_{it}^d, B_{st}^d, R_{st}^d\}\) solves the firm problem (3);
3. \(\{T_t\}\) solves the government problem (1);
4. The production function for new technologies (19) is satisfied; and
5. Markets clear, with
   a. The labor market, \(B_{jt}^s = B_{jt}^d\) and \(R_{jt}^s = R_{jt}^d\) for \(j = i, s\); and
   b. The goods market, \(c_{jt} = Y_{jt}\) for all \(j = i, s\).

3.2. Analytical Results

The firm’s problem is straightforward, but will be useful in the discussion of the household’s problem. Therefore, we start with the firm’s problem.

Firm Optimization

From the firm’s optimization of (3), one can obtain prices for the two final products as well as factor demands for both sectors. The relative wage rates are,

\[
\frac{w_{jbt}}{w_{jrt}} = \alpha_j \left( \frac{M_{bt} B_{jt}}{M_{rt} R_{jt}} \right)^{\frac{\rho - 1}{\rho - 1}}.
\]  
(21)

Using per unit of production factor demands resulting from the relative wage equation and the production function

\[
\frac{B_{jt}}{Y_{jt}} = \frac{1}{Z_t} \left( \frac{\alpha_j M_{bt}^\rho}{w_{jbt}} \right)^{\frac{1}{1 - \rho}} \left[ \alpha_j^{\frac{1}{1 - \rho}} \left( \frac{M_{bt}}{w_{jbt}} \right)^{\frac{\rho}{1 - \rho}} + (1 - \alpha_j)^{\frac{1}{1 - \rho}} \left( \frac{M_{rt}}{w_{jrt}} \right)^{\frac{\rho}{1 - \rho}} \right]^{\frac{1}{\rho}} \text{ for } j = i, s
\]  
(22)

and similarly for \(R_{jt}\), one can obtain the price of the final good \(j = i, s\),

\[
\Rightarrow p_{jt} = \left[ \alpha_j^{\frac{1}{1 - \rho}} \left( \frac{M_{bt}}{w_{jbt}} \right)^{\frac{\rho}{1 - \rho}} + (1 - \alpha_j)^{\frac{1}{1 - \rho}} \left( \frac{M_{rt}}{w_{jrt}} \right)^{\frac{\rho}{1 - \rho}} \right] \frac{\rho}{1 - \rho}.
\]  
(23)
Note, if the two inputs in production are gross substitutes, $0 < \rho < 1$, prices will fall with BBTC (rise in $M_{bt}$; and they will shift more time to the service sector since $\alpha_s > \alpha_i$. However, depending on labor supply effects, it is difficult to tell which sector will have an overall larger price fall.

**Household Optimization**

For a household, the problem is in the same vein as Ngai and Pissarides (2009). Specifically, the household problem can be solved in steps. The household solves three intertemporal choices, starting at the service consumption decision, proceeding to the goods consumption decision, and ending with the leisure decision, plus an intratemporal investment in new technologies decision. Since men process more brawn, they have a comparative advantage in the labor market. Household members consequently specialize, with the man entering the labor market first. As such, we will only analyze the case of $n^m = 0$, i.e., women spend at least a fraction of their time in the labor market. Time subscripts are omitted for all intertemporal decisions.

**Composite Service Consumption**

Households choose time allocated to home production, $n$, in maximizing (8) s.t. (5). The resulting first order condition can be summarized in terms of relative market services to home services,

$$\frac{\hat{c}_s}{c_n} = \left( \frac{a_s}{1 - a_s} \left( \frac{p_n}{(1 - v)p_s + \phi M_n} \right) \right)^{1-\eta},$$

(24)

where $p_n$ is an implicit home production price $p_n = \frac{w_f (1 - \tau)}{M_n}$. As in Ngai and Pissarides (2009), services are “marketized” if $\frac{\hat{c}_s}{c_n}$ rises. The comparative statics for the “marketization” of services, if market and home services are gross substitutes, $0 < \eta < 1$, can be summarized as follows:

- $\frac{\partial \hat{c}_s/c_n}{\partial \tau} < 0$, that is higher taxes discourage market work;
- $\frac{\partial \hat{c}_s/c_n}{\partial w_{sr}} > \frac{\partial \hat{c}_s/c_n}{\partial w_{br}}$, higher brain demand will encourage female market work;

---

8The case with $n^m > 0$ is very similar, however, the implicit home production price will be different in the two cases.
• $\frac{\partial \hat{c}_s}{\partial \phi} > 0$, governments subsidies on, e.g., childcare for working mothers, encourages female market work; and

• $\frac{\partial \hat{c}_s}{\partial p_v} > 0$, a fall in service prices through subsidies or technological progress, especially in brain-augmenting technologies, encourages market work.

**Composite Consumption/Industrial Goods Consumption**

Next households maximize (7), the final composite consumption, by choosing $c_i$. The first order condition can be summarized as relative market service to goods consumption,

\[
\frac{\hat{c}_s}{c_i} = \left( \frac{1 - a_i}{a_i} \frac{p_i}{(1 - v)p_s} a_s \left( \frac{\hat{c}_s}{F(\hat{c}_s, c_n)} \right)^{\eta - \epsilon} \right)^{\frac{1}{1 - \epsilon}}. \tag{25}
\]

Again, we can look at the comparative statics with respect to the key parameters. If services and goods are gross compliments, that is $\epsilon < 0$ and with $0 < \eta < 1$:

• $\frac{\partial \hat{c}_s}{\partial \tau} > 0$, more service marketization leads to rise in relative market service consumption as $\frac{\eta - \epsilon}{1 - \epsilon} > 0$;

• $\frac{\partial \hat{c}_s}{\partial \tau} < 0$, higher taxes, lead to lower service marketization and, therefore, relatively less market service consumption (indirect effect through lower marketization, similarly this will be true for a smaller brain demand);

• $\frac{\partial \hat{c}_s}{\partial \phi} > 0$, again through the indirect effect of marketization, the government can artificially increase the relative service demand; and

• $\frac{\partial \hat{c}_s}{\partial v} > 0$, with a price subsidy on all service goods, the government can increase the relative services demand even further, through both an indirect effect through marketization and a direct price effect.

To summarize the comparative statics between different government actions, if a price subsidy is equivalent to the work subsidy, and the government only employs one at a time, the relative service share will be largest with the price subsidy and smallest without any subsidy, assuming the same tax rate for all
economies, i.e., \( \left( \frac{\hat{c}_s}{c_i} \right)_{\{v>0, \phi=0\}} > \left( \frac{\hat{c}_s}{c_i} \right)_{\{v=0, \phi>0\}} > \left( \frac{\hat{c}_s}{c_i} \right)_{\{v=0, \phi=0\}} \).

The relative market service to industrial goods first order condition, in conjunction with the price equation for final goods, (23), yields some interesting results with respect to innovation (for completeness of the theory, comparative statics for both a rise in \( M_b \) and \( M_r \) are provided). Further wage rates are taken ad hoc. Clearly, while in the general equilibrium these will also affect relative prices, the qualitative aspect of the comparative statics, which is of interest here with respect to technological improvements, stays the same. The partial derivatives are,

\[
\frac{\partial \hat{c}_s}{\partial M_b} = X_b M_r^{\frac{\mu}{1-r}} X_b + X_b \left( M_r^{\frac{\mu}{1-r}} \chi_b + \frac{\eta - \epsilon}{1 - \epsilon} \frac{1}{\alpha_s} \right) \frac{1}{a_s} \left( \frac{c_n}{\hat{c}_s} \right)^\eta, \tag{26}
\]

\[
\frac{\partial \hat{c}_s}{\partial M_r} = X_r M_b^{\frac{\mu}{1-r}} X_r + X_r \left( M_r^{\frac{\mu}{1-r}} \chi_r + \frac{\eta - \epsilon}{1 - \epsilon} \frac{1}{\alpha_i} \right) \frac{1}{a_s} \left( \frac{c_n}{\hat{c}_s} \right)^\eta, \tag{27}
\]

where \( X_b > 0 \) and \( \chi_b = (\alpha_s(1 - \alpha_i))^{\frac{1}{1-r}} - (\alpha_i(1 - \alpha_s))^{\frac{1}{1-r}} > 0 \) and

\[
\frac{\partial \hat{c}_s}{\partial M_r} = X_r M_b^{\frac{\mu}{1-r}} X_r + X_r \left( M_r^{\frac{\mu}{1-r}} \chi_r + \frac{\eta - \epsilon}{1 - \epsilon} \frac{1}{\alpha_i} \right) \frac{1}{a_s} \left( \frac{c_n}{\hat{c}_s} \right)^\eta, \tag{27}
\]

In conclusion, large taxation will lead to a smaller service sector, as fewer women participate in the formal labor market and less services are marke-
tized. The government can affect the relative sector demands by subsidizing consumption of services. However, a subsidy tied to women working is less powerful. Moreover, technical change in brain-intensive technologies will lead to a rise in relative service demand through a direct channel of prices and an indirect channel of home substitution.

**Leisure**

To conclude the intertemporal choices of the household, individuals choose leisure by maximizing (4). The first order condition can be described in terms of relative leisure to consumption,

\[
\frac{\ell}{C} = \frac{1 - a_c}{a_c a_i} \frac{p_i}{w(1 - \tau)(2 - \tau_f) + (1 - v)p_s \phi} \left( \frac{c_i}{C} \right)^{1 - \epsilon}
\]

where \(\ell = \ell^m = \ell^f\). That is, leisure will be greater with government price and work subsidies given equal tax rates.

**Intratemporal Investment**

Lastly, the household makes the intratemporal choice to invest in new brain-augmenting technologies, and solves the dynamic programming problem

\[
\max_{M_b^t} u(C, L) + \beta V(M_b^t),
\]

where \(V(M_b)\) is the value function of the household, given the current technology level is \(M_b\). This problem has a standard Euler equation.

**Sectoral Labor Share**

Using market clearing, the firm and household allocation, the labor supply equations (11), and wages for men and women (9), we can solve for labor shares and wage rates. Specifically, if only men work in the industrial sector, the relative labor allocation is,

\[
\frac{L_s}{L_i} = \frac{c_s p_s^{1 - \rho} \left( \frac{\alpha_s M_b^\rho}{w_i b} \right)^{1 - \rho}}{(c_i + m_b^\rho) \left( \frac{\alpha_i M_b^\rho}{w_i b} + (1 - \alpha_i) M_r^\rho \right)^{1 - \rho}}
\]

and

\[
\frac{L_s}{L_i} = \frac{c_s \left( \alpha_s M_b^\rho + (1 - \alpha_s) M_r^\rho \right)^{1 - \rho}}{(c_i + m_b^\rho) p_i^{1 - \rho} \left( \frac{\alpha_i M_b^\rho}{w_i b} \right)^{1 - \rho}}
\]
if both men and women work in goods production. From the comparative statics on relative market service to goods, marketization will lead to a rise in service labor shares. With larger investment in new technologies relatively more labor will be allocated to the industrial sector, but the effect is temporary. Given wage rates, if only men work in the industrial sector, more brain technology implies larger service labor shares since brain and brawn are gross substitutes, $0 < \rho < 1$. In addition, prices fall with a rise in technology leading to further sectoral reallocation. While it is unlikely, in the context of the model, that women work in both sectors, if they do a rise in brain technologies does not have a direct positive effect on labor shares. Nonetheless, wage rates respond to a change in technology and labor supply, and, therefore, the service labor share might still rise even under this scenario.

Similarly, the wage rates can be solved for a given labor supply of men and women. If men work in both sector, men and women’s labor supply to the service sector will equal

$$L^m_s = \frac{c_s p_s}{\tau_r} \left[ \frac{(1 - \alpha_s) M^\rho_r}{w_{sr}} \right]^{\frac{1}{1-\rho}} - (1 - \tau_r) \left( \frac{\alpha_b M^\rho_b}{w_{br}} \right)^{\frac{1}{1-\rho}}$$

and wage rates using $w_{ir} \equiv 1$ as the numeraire,

$$w_{sr} = \frac{\frac{\alpha_i}{1-\alpha_i} \left( \frac{M_b}{M_r} \right)^\rho + 1}{\frac{\alpha_s}{1-\alpha_s} \left( \frac{M_b}{M_r} \right)^\rho \left( \frac{L^m_s + L^f_s}{L^m_s + (1 - \tau_r)L^f_s} \right)^{\rho-1} + 1}$$

$$w_{sb} = \frac{\alpha_s}{1-\alpha_s} \left( \frac{M_b}{M_r} \right)^\rho \left( \frac{L^m_s + L^f_s}{L^m_s + (1 - \tau_r)L^f_s} \right)^{\rho-1} w_{sr}, \text{ and}$$

$$w_{ib} = \frac{\frac{\alpha_i}{1-\alpha_i} \left( \frac{M_b}{M_r} \right)^\rho}{\frac{\alpha_s}{1-\alpha_s} \left( \frac{M_b}{M_r} \right)^\rho}.$$
Similarly, if women work in both sectors,

\[
L_i^m = \left( c_i + m_b' \right) \frac{1}{\rho} \left[ \left( \frac{1 - \alpha_i}{w_i r} \right)^{\frac{1}{\rho}} - \left( \frac{\alpha_i M_b^\rho}{w_i b} \right)^{\frac{1}{\rho}} \right], \quad (38)
\]

\[
L_i^f = \left( c_i + m_b' \right) \frac{1}{\rho} \left[ \frac{\tau_r}{\rho} \left( \frac{\alpha_i M_b^\rho}{w_i b} \right)^{\frac{1}{\rho}} - \left( \frac{(1 - \alpha_i) M_f^\rho}{w_i r} \right)^{\frac{1}{\rho}} \right],
\]

and wages are determined by

\[
w_{sr} = \frac{\alpha_i}{1 - \alpha_i} \left( \frac{M_b}{M_r} \right)^\rho \left( \frac{L_i^m + L_i^f}{L_i^m + (1 - \tau_r)L_i^f} \right)^{\rho-1} + 1 - \tau_r, \quad (39)
\]

\[
w_{sb} = \frac{\alpha_s}{1 - \alpha_s} \left( \frac{M_b}{M_r} \right)^\rho \left( \frac{1}{1 - \tau_r} \right)^{\rho-1} w_{sr}, \quad (40)
\]

\[
w_{ib} = \frac{\alpha_i}{1 - \alpha_i} \left( \frac{M_b}{M_r} \right)^\rho \left( \frac{L_i^m + L_i^f}{L_i^m + (1 - \tau_r)L_i^f} \right)^{\rho-1}.
\]

As expected, innovation in brain technologies leads to a rise in brain wage rates. However, there is also a supply effect, as brain to brawn labor supply shares rise, lowering the wage rate of brain in the service sector when men work in both sectors and in the industrial sector otherwise. The effect on the brawn wage rate in the service sector is slightly more complicated but a general rise in brain technologies will lower brawn wage rate, since \( \alpha_s > \alpha_i \).

In summary, this section has shown that larger government taxation will lead to a smaller service sector and less female labor force participation. Governments can increase female labor force participation, and therefore also the service labor share, by subsidizing female employment or the purchase of services. In addition, subsidizing female employment through a rebate in services has the added effect of increasing service employment, and therefore brain demand, e.g., more childcare facilities do not only provide services for households, but also employment opportunities for women.
4. Simple Calibration

The model is simulated to the United States, tax rates are then adjusted to account for differences between Continental Europe, Nordic countries, and the United States. Tax rates are taken from McDaniel (2006). Although tax rates have been increasing over time in all three regions, this calibration takes an average tax rate over time, with the United States having a 22 percent tax rate, and Europe and the Nordic countries having a 43 percent tax. The parameter governing the elasticity of substitution between home and market services, $\eta$, and the elasticity between goods and services, $\epsilon$, are taken from previous studies. Various studies have estimated $\eta$ on microeconomic and macroeconomic data. The resulting elasticities vary from 1.60 to 2.00 by Rupert et al. (1995), depending on the whether households are married, single females or single males, to 2.3 by Chang and Schorfheide (2003). Aguiar and Hurst (2006) find an elasticity of 1.80, which implies an $\eta$ of 0.45 used in this calibration. Ngai and Pissarides (2009) suggest that, given price elasticities of the entire service sector of $-0.30$ to $-0.06$, a model with home production the elasticity of 0.30 should be an upper bound, implying a value of $\epsilon = -2.30$, which is used in the calibration below.

To estimate the elasticity in firm production, we use the 1977 and 1991 DOT factor estimates to construct an aggregate data series of brain and brawn supply over time (see Appendix A for details). The elasticity is assumed to be equal across sectors. To determine the substitution parameter, $\rho$, taking logs of (21), we can estimate a regression similar to Katz and Murphy (1992, pg. 69), where the constructed brain to brawn aggregate are the relative labor supplies. The wage series of brain and brawn prices is constructed by estimating the regression of average residual male wages by occupation over time on brain and brawn factors, capturing occupation specific returns (see Appendix A for details). The residual wages are taken from a Mincer-wage regression of full-time male workers\footnote{Full-time workers are defined as working at least 39 weeks and 35 hours per week (prior to 1976 only hours worked prior to the survey week are recorded).} on standard explanatory variables. The resulting estimate of $\rho$ varies between 0.62 for the 1977 DOT and 0.67 for the 1991 DOT, therefore $\rho$ is set to 0.65.

The brawn factor $M_r$ is normalized to one. The yearly discount rate
is set to $\beta = 0.96$. Home productivity $M_n$ is assumed constant over time, abstracting from improvements in home technology, which could partly explain the rise in female employment. It is assumed that the United States has reached its steady state by 2005 (there is evidence of flattening female labor force participation and gender wage difference in the last few years). Therefore, the investment technology must reach the steady state within 56 years from the 1950 starting point, helping to pin down $\psi_m$. The remaining parameters $\{\alpha_s, \alpha_i, a_c, a_s, a_i, M_{b1950}, M_n, \tau, \delta\}$ are set in order to match the following nine steady state (2005) and 1950 targets:

- Gender wage gap;
- Relative goods to services hours;
- Married female market hours of work;\(^\text{10}\)
- Married male market hours of work; plus
- Growth rate in male wages from 1950 to 2005.

The model does well in matching all targets, except it does underestimate the rise in female hours worked by four percentage points. Specifically, while female hours relative to total hours worked rose from about six to 23 percentage points, the model generates a rise from six to 19 points. The calibrated parameters are summarized in table 2.

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Value</th>
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<tr>
<td>$\alpha_s$</td>
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</tr>
<tr>
<td>$\alpha_i$</td>
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<tr>
<td>$a_c$</td>
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<td>$a_i$</td>
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<td>$a_s$</td>
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<td>$\tau_r$</td>
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<td>$M_{b1950}$</td>
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<tr>
<td>$M_{n1950}$</td>
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</tr>
<tr>
<td>$\delta$</td>
<td>.36378</td>
</tr>
<tr>
<td>$\psi_m$</td>
<td>-.14537</td>
</tr>
</tbody>
</table>

\(^{\text{10}}\) Assuming individuals have a total time endowment of $15 \times 365$ hours
4.1. Results

The results presented in this section show that the mechanism highlighted in this study does well in generating differences in female employment rates and service sector labor shares across different tax regions. However, given that innovation is directly linked to the goods sector, the model is unable to produce the same sharp fall and initially larger goods sector for Europe as in the data (see figure 4). The relative service share and female employment share could be partially underestimated since, for example, France does provide some childcare provision for working mother beyond what is offered in the United States.

For the Scandinavian countries, we do not have a good estimate of what $\phi$ should be. However, we can calibrate $\phi$ such that the relative share of female employment is identical to the data in 2005, i.e., the relative female labor share of the Nordic countries to the United States was about 95 percent.
in 2005. In this case, $\phi = 3.50$, which equates in 2005 to about 22 percent of the market services consumed are provided by the government through female employment subsidies, which accounts to a social welfare loss in terms of utils of roughly four percent to up to 12 percent in the earlier stages of development.

Moreover, figure 5 shows the evolution of the United States gender wage gap, since the calibration fixes both 1950 and 2005 wage gaps it is not surprising it matches the overall trend. However, the model also does well in matching the evolution over time. We first observe a period of wage stagnation, albeit shorter than in the data, and than a steady closing of wages until it levels off. The effects contributing to the shape are the changes in relative wage rates, because of technological innovation and changing relative brain and brawn supplies. As more women enter the labor market, relative brain to brawn supplies rise disproportionately and can therefore lead to a stagnation of relative wages. Finally, as the economy invests less in brain-augmenting
new technologies, the wage gap levels off.

Lastly, to illustrate the importance of women’s lower brawn abilities, Figure 6 shows relative female hours compared to the base case of low taxation with the original calibrated productivity difference $\tau_r$. While less productivity differences between men and women initially lead to a larger female labor force participation, the trend is reversed over time. Technological innovation in the benchmark case is more important to diminish women’s productivity differences. Therefore, at the beginning of “development” women stay at home, but as the economy has a greater incentive to innovate it does so leading to greater feedback effects and eventually, women employment becomes larger in the base case.
5. Conclusion

To summarize, this paper has developed a theory that can explain both a rise in services and a rise in female labor supply concurrently. It accounts well for differences between Continental Europe and the United States in the service sector labor share and female employment over time. However, given the technology to innovate in brain-augmenting factors, the model is unable to match the larger goods sector in Europe around the 1950s. In this study we have assumed Europe and the United States are on the same development level. Rogerson (2007a) argues that Europe in the 1950s was behind the United States in the development phase and was catching up towards 2000. Clearly, this could partially explain a larger goods sector. In general, the model is consistent with the hypothesis that a high tax country will have a smaller service sector, less innovation in brain-intensive technologies, and less female labor force participation. A simple computation of the social welfare loss of high taxes and subsidizing female labor force participation in Scandinavia compared to an equivalent outcome in female labor force participation with low taxes and no subsidies shows a non-negligible loss. Moreover, the model is not only able to replicate some cross-region differences, but also the evolution of the wage gap in the United States, matching part of the early stagnation in the gender wage gap.
References


Appendix A: Factor Estimation

[To be added]