Essays on Dynamic Macroeconomics

Ceyhun Elgin
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Contents

Acknowledgements i

Abstract vi

Contents vii

1. Political Turnover, Taxes, and the Shadow Economy 1

2. A Theory of Economic Development with Endogenous Fertility 52

3. Not-Quite-Great Depressions of Turkey 80
Essays on Dynamic Macroeconomics

A Dissertation Submitted to the Faculty of the Graduate School of
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by

Ceyhun Elgin

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Institute.

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Unfortunately, my dear grandfather M. Cezmi Elgin and my dear father M. Akın Elgin could not live to see me receive my Ph.D. I am sure they would have been very proud of me.
I dedicate this dissertation to my beloved family: My dear mother Füsun Elgin, my father M. Akın Elgin, my grandfathers H. Basri Akgiray, M. Cezmi Elgin, and my uncle Attila Kurtaran who in the absence of my father were like fathers to me and last but not least my dear girlfriend Cansu Yüksel.

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Abstract

This dissertation, titled "Essays on Dynamic Macroeconomics" is comprised of three papers which share the common ground of employing tools of modern dynamic macroeconomics. All the three papers apply modern technical tools of macroeconomics in dynamic environments to different macroeconomic problems and observations, develop mechanisms and models to account for these observations and finally test explanatory powers of the provided mechanisms.

The first paper, titled "Political Turnover, Taxes, and the Shadow Economy" is motivated from several cross-section empirical studies which argue that a higher tax burden or different indicators of statutory tax rates are associated with a smaller informal economy. In the paper I show that the turnover of governments provides the key to understanding this relation. To this end, I present evidence that once political turnover is controlled for, the data shows no association between the tax burden and the size of the informal economy. This result is empirically robust in a panel data consisting of 80 countries and 5 years. To account for this observation, I develop a dynamic political economy model with two political parties alternating in office. In equilibrium, if the incumbent party faces a higher probability of staying in office, it sets a higher tax rate to invest more in productive public capital, while spending less for current office rent. I argue that public capital is mainly utilized by the formal sector and this implies that countries in which incumbent parties are more likely to stay in power, have a higher tax burden but a smaller informal sector. Finally, I compare the model against the data and present evidence that my theory is consistent with empirical
observations.

In the second paper, titled "A Theory of Economic Development with Endogenous Fertility", I integrate two existing theories of economic development to account for the time-series evolution of output, fertility and population in transition through the industrialization of an economy. Specifically, I extend a standard two-sector overlapping generations model with endogenous fertility and human capital decisions. Initially, the aggregate human capital and return to education are low and parents invest in quantity of children. Once sufficient human capital is accumulated, with the activation of the modern human capital intensive sector, parents start to invest in quality of their children. The simulation of the model economy successfully captures the evolution of fertility, population and GDP of the British economy between 1750 and 2000.

Finally, in the third chapter, titled "Not-Quite-Great Depressions of Turkey", which is based on a paper written together with my coauthor Deniz Çiçek, following the great depressions methodology we use growth accounting and perfect foresight dynamic general equilibrium models to study growth performance of Turkey from 1968 to 2004. We calculate the total factor productivity from the growth accounting exercise and obtain the consumption tax rate and the effective marginal tax rates on labor and capital income from the data and feed them into our model. Our benchmark model without any frictions and taxes accounts for 86% of the observed change in the growth rate of GDP per-working age person from 1968 to 2004 and once we extend the model with taxes and capital adjustment costs it accounts for 60% of the observed reduction in hours worked per-working age person and 35% of the change in the growth rate of capital-output ratio from 1968
to 2004. Also, we identify that the Turkish economy experienced a depression from 1976 to 1984 and the extended model performs remarkably well to account for the observed change in GDP per-working age person, capital-output ratio and hours worked during this depression period. Our findings generally suggest that rigidities affecting capital accumulation and government policies using distortionary taxes have a crucial role in explaining the evolution of the selected variables of the Turkish economy.
Contents

Front Matter i

Acknowledgements .................................. iii
Dedication ......................................... iii
Abstract ............................................ vi

1 Political Turnover, Taxes, and the Shadow Economy 1

1.1 Introduction ...................................... 1
  1.1.1 Motivation .................................. 1
  1.1.2 Contribution of This Paper ................. 7
  1.1.3 Road Map .................................. 8
1.2 What Do Data Tell? ............................. 9
  1.2.1 Taxes and the Informal Sector .............. 9
  1.2.2 Do the Data Tell More? ..................... 19
1.3 Model .......................................... 23
  1.3.1 Households ................................ 24
1.3.2 Technology ............................................. 26
1.3.3 Government ........................................... 27
1.3.4 Competitive Equilibrium .............................. 29
1.3.5 Politico-Economic Equilibrium ....................... 31

1.4 Numerical Analysis ........................................ 45
1.4.1 Parametrization and Calibration ....................... 45
1.4.2 Quantitative Results and Experiments ............... 46

1.5 Conclusion ............................................... 49

2 A Theory of Economic Development with Endogenous Fer-
tility .......................................................... 52
2.1 Introduction .............................................. 52
2.2 Empirical Motivation ...................................... 58
2.3 The Model .................................................. 63
   2.3.1 Households’ Problem ............................... 63
   2.3.2 Technology ......................................... 65
   2.3.3 Equilibrium and Characterization .................. 67
2.4 Simulation .................................................. 71
2.5 Concluding remarks ....................................... 78

3 Not-Quite-Great Depressions of Turkey .................. 80
3.1 Introduction .............................................. 80
3.2 Evolution of the Turkish Economy ....................... 84
### Table of Contents

3.2.1 Inspecting the GDP data ........................................... 84
3.2.2 Growth Accounting .................................................. 88
3.3 The Dynamic General Equilibrium Model ............................... 93
  3.3.1 The Benchmark Model .............................................. 93
  3.3.2 Adding adjustment costs to capital accumulation ............. 96
  3.3.3 Adding taxes ...................................................... 97
  3.3.4 Complete Model .................................................. 99
3.4 Numerical Experiments .................................................. 99
  3.4.1 Calibration ....................................................... 99
  3.4.2 Simulation Results .............................................. 103
3.5 Conclusion .................................................................. 107

4 Appendix .................................................................. 113
  4.1 Appendix A: Appendix to Chapter 1 ................................. 113
    4.1.1 Proof of Proposition 1.3.3 ...................................... 113
    4.1.2 Proofs of Propositions 1.3.4 and 1.3.5 ....................... 118
    4.1.3 Computational Algorithm ...................................... 124
    4.1.4 Country List .................................................... 126
  4.2 Appendix B: Appendix to Chapter 2 ................................. 127
  4.3 Appendix C: Appendix to Chapter 3 ................................. 130

Bibliography .................................................................. 132
List of Tables

1.1 Informal Sector and Tax Burden .......................... 17
1.2 Informal Sector and Fiscal Freedom ....................... 18
1.3 Summary Statistics ........................................ 20
1.4 Regressions with Political Turnover ...................... 22
1.5 Informal Sector and Political Turnover ................... 23

2.1 Values for Basic Parameters .............................. 72

3.1 Data and the model without adjustment costs .............. 102
3.2 Data and the model with adjustment costs ................. 103
List of Figures

1.1 Informal Sector vs. Tax Burden ................. 13
1.2 Informal Sector vs. Income Taxes ................ 14
1.3 Informal Sector vs. Statutory Taxes .............. 15
1.4 Informal Sector vs. Probability of Reelection .......... 47
1.5 Tax Burden vs. Probability of Reelection ........... 47
1.6 Public Investment vs. Probability of Reelection ...... 48
1.7 Office Rent vs. Probability of Reelection ........... 48
1.8 Informal Sector and Tax Burden: Data vs. Model ........ 50
1.9 Informal Sector and Tax Burden: Data vs. Model Regressions 50

2.1 Population ........................................ 60
2.2 Population Growth ................................... 60
2.3 Gross Reproduction Rate and Average Life Expectancy ... 61
2.4 GDP and GDP per-capita ............................ 61
2.5 Population and Population Growth: Data vs. Model .... 75
3.11 Detrended real GDP per person in Turkey: Data and model simulations ............................................. 109
3.12 Capital/output ratio in Turkey: Data and model simulations ......................................................... 109
3.13 Hours worked per person in Turkey: Data and model simulations .................................................. 110
3.14 Detrended real GDP per person in Turkey: Data and model simulations (with adjustment costs) ............. 110
3.15 Capital/output ratio in Turkey: Data and model simulations (with adjustment costs) .............................. 111
3.16 Hours worked per person in Turkey: Data and model simulations (with adjustment costs) ..................... 111
Chapter 1

Political Turnover, Taxes, and the Shadow Economy

1.1 Introduction

1.1.1 Motivation

Several cross-section and panel data empirical studies associate higher tax rates with a smaller informal economy\(^1\). Examples of such studies are Johnson et. al. (1997, 1998), Friedman et. al. (2000), and more recently

\(^1\)Informal economy or informal sector, sometimes also called as shadow, hidden or underground economy is defined by Hart (2008) as a set of economic activities that take place outside the framework of bureaucratic public and private sector establishments. Another paper by Ihrig and Moe (2004) defines it as a sector which produces legal goods, but does not comply with government regulations.
Torgler and Schneider (2007)\textsuperscript{2}. Graphically, plotting informal sector size vs. tax burden, corporate tax rate, average labor income tax rate, or top marginal income tax rate\textsuperscript{3} in a cross-section clearly indicates a negative relationship between these variables.

In this paper, I first employ cross-section, static and dynamic panel data techniques to show that the negative relationship between various measures of tax rates and the size of the informal sector is significant and robust. Moreover, the econometric analysis also explores what factors might have caused it. To this end, I present evidence that once political turnover is controlled for, the data shows no significant association between tax rates or tax burden and the size of the informal economy. Next, building upon the empirical analysis, I develop a dynamic political economy model to account for this observation. In the model, the government that lacks the ability to commit to future policy choices uses taxes on capital and labor income of the formal sector to finance the provision of a productive public capital and some office rent. The government is not fully benevolent and also gets utility from some office rent, the amount of which is chosen by the incumbent.

\textsuperscript{2}More recently, Aruoba (2009) also documents a negative correlation between taxes and the size of the informal economy.

\textsuperscript{3}At this point it may be important to emphasize the distinction between the tax burden and various statutory tax rates. Tax burden is defined as the ratio of total tax revenues to GDP and one might suspect that the negative relation between the tax burden and the informal sector may arise simply because a larger informal economy implies a smaller tax base, thereof a lower level of tax revenue. However, considering that only imperfect estimates of the informal economy are included in the national income calculations, a larger informal economy also implies a lower level of official GDP. Moreover, as the empirical analysis in the next section clearly shows, the negative relation is also evident between various statutory tax rates and the size of the informal sector.
government. Then I introduce political frictions to the model, specifically by allowing two political parties to alternate the office with some exogenous probability (i.e. Incumbency follows a simple Markov chain.), and focus on the symmetric differentiable (interior) Markov perfect equilibrium of this environment. In equilibrium, if the incumbent party faces a higher probability of keeping the office (i.e. the lower the political turnover), it has higher stakes in the future (because probability of enjoying future office rent is higher) and it values future output more. Therefore, it charges a higher tax rate today on the formal sector to invest more on productive public capital, while spending less for current office rent. This result is based on the fact that a higher probability of keeping the office next period (i.e. the incumbent gets more certain of it’s tenure) changes the marginal rate of substitution between future office rent and current office rent and therefore the incumbent spends less for the office rent today (i.e. steals less today) and invests more in the productive public capital of tomorrow. Even though the tax burden is higher, the tax revenue is increasingly used for the productive public good in the formal sector. This stimulates incentives for being formal and reduces the size of the informal sector. This result captures the main empirical findings of the above mentioned papers and my empirical analysis.

As described above, the model suggests that political frictions, more specifically political turnover affecting corruption (office-rent in my model’s terms ) and the provision of a productive public capital in the formal sector
are among the underlying causes of the negative relationship between taxes and size of the informal sector. In the last part of the paper, I compare the implications of the model against the data. Specifically, I take the exogenously given probability of reelection data from a recent paper by Brender and Drazen (2008), feed them into the model, and then compare various variables of interest generated by the model against their counterparts in the data. Once calibrated to match certain specific moments, the model performs quite well to account for the cross-country correlation between the tax burden and the size of the informal sector.

My paper is distinct in the growing literature on the informal sector. As opposed to the above mentioned empirical analyses, a common result in models dealing with an informal sector is a positive relationship between the level of tax rate and the size of the informal sector. A non-exhaustive list of the papers in this literature include Rauch (1991), Loayza (1996), Fortin et.al (1997), Ihrig and Moe (2004), Busato and Chiarini (2004) and Amaral and Quintin (2006). This result seems to be intuitive because higher tax rates may create incentives for people to avoid them and one way of doing this is participating in the informal sector. Keeping taxes exogenous and letting the informal sector not paying any taxes (or letting it pay a smaller fraction than the formal sector), this result is also immediate in a two-sector neoclassical growth model with formal and informal sectors, where the variation in taxes in exogenous. An alternative theoretical possibility
might be that a higher tax rate results from some institutional frictions (such as a low degree of tax enforcement) which may create a larger informal sector and therefore, a smaller formal sector tax base. Following this reasoning, in a two-sector environment with a benevolent government which taxes the formal sector to finance some exogenous stream of government expenditures, a Ramsey equilibrium features a positive relationship between tax rates and the size of the informal sector, i.e. a larger informal sector resulting due to some friction (i.e. lower tax enforcement, or lower productivity gap between the formal and the informal sectors) leads to a higher tax rate in the formal sector. So existing theoretical frameworks cannot account the somewhat surprising negative relationship between tax rates and the size of the informal sector.

Some of the above mentioned empirical papers indicating a negative relationship between tax rates and the informal sector deserve more discussion as they are more closely related to my paper.

Both Johnson et. al (1997) and Johnson et. al (1998) use different sets of countries in their empirical analyses; however, both end up with the conclusion that tax rates are negatively correlated with the size of the informal sector. Johnson et. al (1997) also provide a very simple model in which the only two stable equilibria of the model feature totally formal and totally informal economy. However, their model, contrary to their empirical findings, implies a positive relationship between the tax rates and the size
of the informal sector. On the other hand, Johnson et al. (1998) claim that both administration of taxes and regulatory discretion are playing key roles in this result and once they take composite indices of both tax rates and quality of tax administrations into account, they find that these indices are positively correlated with the size of the informal sector. However, the quality indices they use are largely based on subjective evaluations of certain experts and institutions and therefore prone to measurement errors and endogeneity issues.

Friedman et al. (2000) suggest that the positive correlation might have been caused by several institutional factors such as corruption and bureaucratic quality. Accordingly, these factors could let the businesses hide their activities from the government, which by reducing the tax revenues and harming the quality of public administration further reduces a firms incentives to remain formal. In their empirical study, they also find that increasing tax rates by one point implies that the share of the unofficial economy falls by 9.1%. Controlling for several variables and instrumenting on others reduces this number by half, but the negative tax coefficient remains significant. The conclusion of their empirical study is that this is probably because higher tax rates generate revenue that provides productivity enhancing public goods, a strong legal environment and low corruption. However, they only consider the production side of the economy and their highly stylized partial equilibrium model only focuses on the corruption part
of the story.\textsuperscript{4}

The modeling of public finance in my paper is related to the growing literature of Markov-perfect taxation models. Earlier work in this literature includes Cohen and Michel (1988) and Currie and Levine (1993). Later, Klein and Rios-Rull (2003) analyzed Markov-perfect labor and capital taxes in a model where the government can only commit to the following period’s capital tax. More recently, Klein, Krusell and Rios-Rull (2008) and Martin (2009) study a model of public expenditure and characterize and solve for the equilibrium of the dynamic game between successive governments. As opposed to my work, none of the above mentioned papers have a political economy dimension or an informal sector.

1.1.2 Contribution of This Paper

This paper contributes mainly to the literature on informal economy and taxes, informal economy and corruption, and informal economy and productive public goods in three dimensions. First, noticing that most work done in these areas are empirical and lack a strong theoretical basis, this paper provides a general equilibrium model and fills in the theoretical gap in the literature with a novel mechanism. Second, to the best of my knowledge, this paper is one of the few attempts to utilize empirical results of a panel

\textsuperscript{4}In the next section of my paper I show that the negative correlation between taxes and informal sector remains significant, even after controlling for corruption. This suggests that corruption only does not explain this phenomenon.
data set among the set of empirical papers on the informal sector. Lastly, this paper also contributes to the literature on optimal Markov-perfect fiscal policy by adding an informal sector and a political economy dimension to standard models of this literature.

1.1.3 Road Map

The rest of the paper is organized as follows: Empirical evidence indicating a robust negative relationship between taxes and the informal sector is provided in the next section. In this section, I also provide some empirical support to motivate the political turnover’s role in the mechanism of the paper. Specifically, I empirically investigate what causes the negative relationship between taxes and the size of the informal sector. In section 3 the benchmark model is presented. Here, I first describe the environment and then define and characterize the competitive equilibrium. Next, given the competitive equilibrium, the symmetric differentiable Markov-perfect equilibrium is defined and characterized. Section 4 describes empirical implications of the model and then compare model simulations against the data. Lastly, section 5 concludes.
1.2 What Do Data Tell?

This section presents the relevant data and empirically investigates the relationship between the size of the informal sector and various measures of tax rates. In the first part of this section I show that the negative relationship between taxes and the size of the informal sector is robust. Then in the second subsection I investigate what factor might be causing this surprising result and present empirical evidence to motivate the main mechanism of the model.

1.2.1 Taxes and the Informal Sector

This subsection investigates the relationship between different measures of taxes and the size of the informal sector. First, I describe the data and then present results of several econometric estimations.

Data

**Informal Sector Size:** The informal sector consists of economic activities that are not reported to the government statistical offices. Statistical offices usually try to estimate these activities in the unofficial economy; however, these estimations are imperfect by their nature. In the literature people used various methods to estimate the size of the informal sector in a given economy. One method is exploiting the fact that the short-run electricity-
to-GDP elasticity is usually close to one and uses electricity consumption to estimate the informal sector size. An alternative method is the MIMIC (multiple-indicator multiple-cause) approach in which the size of the informal economy is estimated from observations of the likely causes and effects of the underground economy. Lastly, there is also the currency demand approach which is based on demand for cash-to-GDP elasticity, similar to the electricity consumption. Obviously, each method has its own advantages and disadvantages the discussion of which is out of the scope of this paper. In this paper, I use panel estimates of Schneider (2007) running from 1999 to 2005 which combines a dynamic version of MIMIC with the currency demand approach.

**Taxes:**

In the econometric estimations I use various measures of taxes to check the robustness of the analysis. One such measure is the tax burden data from the Government Finance Statistics (GFS) data of IMF. I also use taxes on income, profits and capital gains (as percentage of GDP) from the World Development Indicators. Moreover, I also used the fiscal freedom indicator.

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5 See Kaufman and Kaliberda (1996) for details of this method.
6 MIMIC method is first suggested by Loayza (1996).
7 See Tanzi (1999) and Schneider (2007) for a discussion.
8 Schneider (2007) reports one estimate for two consecutive years, so the span of the time series is 5.
9 See Schneider (2007) for details and superiority of this methodology to others and comparisons of various methods previously used to estimate the size of the informal sector.
10 Throughout this paper tax burden is defined as the ratio of total tax revenues to GDP.
of the Heritage Foundation which is a composite index stemming from the
top tax rate on individual income, the top tax rate on corporate income, and
total tax revenue as a percentage of GDP. Yet another alternative source is
the data on top marginal income tax rate from the Fraser Institute. The
reported regression results mainly use the tax burden data from the GFS;
however, the results do not change if one uses other types of taxation data
from the above mentioned sources.\textsuperscript{11} Notice that results also do not depend
on whether one uses data on statutory taxes (main part of the Heritage
Foundation’s fiscal freedom index) or actual taxes, such as the tax burden
data from GFS.\textsuperscript{12}

To illustrate the negative correlation, figure 1 depicts the relationship
between the informal sector size and tax burden in a cross-section. Figure
2 uses the ratio of revenue from taxes on income, capital gains and profits
to the GDP on the x-axis. Moreover, figure 3 draws\textsuperscript{13} informal sector size
vs. the fiscal freedom index provided by the Heritage Foundation.\textsuperscript{14}

One can also argue that a large (small) informal sector resulting in a
small (large) formal sector; therefore, a small (large) tax base could lead
to a low (high) level of tax revenue and therefore reduce (increase) the

\textsuperscript{11}Estimation results using the various different tax data are available upon request.
\textsuperscript{12}Also see Aruoba (2009) for a discussion.
\textsuperscript{13}All figures use cross-section averages for 80 countries between the years 1999 and 2005. The list of these 80 countries is provided in appendix 6.4.
\textsuperscript{14}Notice that the freedom index gets higher values when tax rates get smaller, therefore a positive correlation between the index and informal sector size is qualitatively equivalent to a negative correlation between taxes and the size of the informal sector.
tax burden which makes the informal sector size and the tax burden to be negatively correlated. However, since the official GDP statistics include only imperfect estimates of the informal sector, a large informal sector also reduces the official GDP which is the denominator in the tax burden formula. Moreover, in case official GDP statistics include perfect estimates of the informal sector size I also check the correlation between the informal sector size and a different measure of the tax burden, by dividing the total tax revenue not by GDP but instead to GDP subtracted by the total informal sector size. The correlation between this measure of the tax burden and the size of the informal sector is $-0.46$. This indicates that the negative correlation between the tax burden and the size of the informal sector does not arise from a variable tax base depending on the size of the formal sector.

Other variables:

In the regression analysis I also use several control variables, such as GDP per-capita, corruption and bureaucratic quality. I got the data for GDP per-capita from the Groningen Economic Growth and Development Center. For corruption, I use corruption index data both from Transparency International and Political Risk Services (ICRG). Similarly, the measure of bureaucratic quality is obtained from ICRG, too. These three variables are the ones extensively used in the empirical literature on the causes of the informal sector.

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15Reported results use data from ICRG, however using Corruption Perceptions Index from Transparency International do not change the results of the estimations.
Estimation and Results

There are a number of studies analyzing the empirical relationship between taxes and the size of the informal sector. In certain studies, especially those who do not control for variables measuring institutional quality, found some empirical support suggesting a positive relationship between taxes and the shadow economy. Schneider and Enste (2000) provide an excellent review of this empirical literature. However, other empirical studies such as Johnson et.al. (1998), Friedman et.al (2000), Kucera and Xenogiani (2009) revealed that, once institutional quality is taken into account, the size of the
informal sector and various measures of tax rates are negatively correlated.

To check the robustness of the negative relationship evident in figures 1.1, 1.2 and 1.3, I run a number of regressions using different explanatory variables.

In the static panel data analysis\textsuperscript{16}, the estimated equations are of the following form:

\[ IS_{i,t} = \beta_0 + \beta_1 \text{tax}_{i,t} + \sum_{k=2}^{n} \beta_k \text{X}_{k_{i,t}} + \theta_i + \gamma_t + \epsilon_{i,t} \]

\textsuperscript{16}I also report results of a cross-section estimation using the 5-year averages of the panel data.
where $X_{k_{i,t}}$ are the other explanatory variables in addition to taxes and $\theta_t$, $\gamma_t$ are the country and period fixed effects, respectively. Moreover $IS_{i,t}$ is the size of the informal sector relative to GDP and $tax_{i,t}$ is the tax rate. Notice that, when I include institutional variables such as corruption, and bureaucratic quality in $X_{k_{i,t}}$ (and to some extent even the GDP per-capita) the estimation may become prone to endogeneity issues. Therefore, I also redo the estimation using instrumental variables, namely latitude (Hall and Jones (1999)), an indicator variable for presidential vs. parliamentary regimes (Leiderman et. al. (2005)), an indicator variable for transition countries, and
indicator variables for the legal system (La Porta et al. (1999)).

One should also notice that, in addition to the cross-country pooled regression and the static panel data analysis, I also perform a dynamic panel data analysis in which I use one-period lagged value of the informal sector size as an additional independent variable. Specifically, I estimate the following equation:

\[ IS_{i,t} = \beta_0 + \beta_1 tax_{i,t} + \beta_2 IS_{i,t-1} + \sum_{k=3}^{n} \beta_k X_{k,i,t} + \theta_t + \gamma_t + \epsilon_{i,t} \]

Static panel data models and their estimators do not take the serial correlation, heteroscedasticity and endogeneity problems that may occur in such dynamic models into account. To overcome these kind of problems, dynamic panel data model estimation techniques a la Anderson and Hsiao (1981) and Anderson and Hsiao (1982) can be used since they were first to develop an instrumental variables technique to estimate dynamic models. Then, as well known, Griliches and Hausman (1986), Holtz-Eakin et al. (1988) also developed similar estimators. These estimators use lagged values of the dependent variable as instruments in the differenced equations. They are consistent but generally not efficient since they do not take all restrictions on the covariances between regressors and the error term into account. To overcome this issue, Arellano and Bond (1991) developed a dynamic version of the generalized method of moments estimator. They argued that the
Table 1.1: Informal Sector and Tax Burden

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<td>-1.8</td>
<td>-1.78</td>
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<td></td>
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<td>(0.76)</td>
<td>(0.73)</td>
<td>(0.79)</td>
<td>(0.74)</td>
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<tr>
<td>Corruption</td>
<td>-1.7</td>
<td>-1.4</td>
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<td>-1.41</td>
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<tr>
<td></td>
<td>(0.67)</td>
<td>(0.65)</td>
<td>(0.69)</td>
<td>(0.63)</td>
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<tr>
<td>IS(-1)</td>
<td>0.39</td>
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<td></td>
<td>(0.12)</td>
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</tbody>
</table>

*R-squared* | 0.57 | 0.35 | 0.41 | 0.44 | 0.45 |
*Observations* | 80 | 400 | 400 | 400 | 240 |
*F-Test* | 29.18 | 78.71 | 49.52 | 63.38 | |
*Hansen J-Test* | 0.11 | |
*AR(2) Test* | 0.28 | |

All panel regressions include year and country fixed effects. Standard errors are reported coefficient in parentheses.

estimators obtained through this method are also efficient since this method is based on using additional instruments (lagged values of the dependent variables and other explanatory variables) which satisfy the orthogonality conditions.

All the results of the above described estimations are presented in table 1.1. First columns present the cross-section regression results whereas second, third and the fourth columns show the fixed effect panel data regression outputs. In column 5 I report the results of the IV estimation and lastly in the sixth column I present the results of the dynamic panel data analysis using the above discussed Arellano-Bond GMM estimation. The results indicate that that negative relationship between the tax burden and the size of the informal sector is quite robust. Moreover, in table 1.2, analogous to
Table 1.2: Informal Sector and Fiscal Freedom

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</tr>
</thead>
<tbody>
<tr>
<td>Fiscal Freedom Index</td>
<td>0.14 (0.059)</td>
<td>0.17 (0.06)</td>
<td>0.12 (0.05)</td>
<td>0.19 (0.06)</td>
<td>0.23 (0.05)</td>
<td>0.18 (0.04)</td>
</tr>
<tr>
<td>GDP per-capita</td>
<td>-0.093 (0.025)</td>
<td>-0.075 (0.024)</td>
<td>-0.077 (0.026)</td>
<td>-0.079 (0.029)</td>
<td>-0.081 (0.031)</td>
<td>-0.082 (0.034)</td>
</tr>
<tr>
<td>Bureaucratic Quality</td>
<td>-1.84 (0.75)</td>
<td>-1.74 (0.78)</td>
<td>-1.76 (0.91)</td>
<td>-1.74 (0.89)</td>
<td>-1.68 (0.92)</td>
<td></td>
</tr>
<tr>
<td>Corruption</td>
<td>-1.76 (0.67)</td>
<td>-1.44 (0.62)</td>
<td>-1.41 (0.69)</td>
<td>-1.42 (0.65)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>IS(-1)</td>
<td>0.40 (0.40)</td>
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</tr>
</tbody>
</table>

$R^2$-squared | 0.52 | 0.33 | 0.39 | 0.44 | 0.46 |
Observations | 80 | 400 | 400 | 400 | 240 |
F-Test | 21.45 | 71.84 | 44.18 | 58.67 |
Hansen J-Test | 0.09 |
AR(2) Test | 0.23 |

All panel regressions include year and country fixed effects. Standard errors are reported coefficient in parentheses.

table 1.1, I report the results when I use the fiscal freedom index, instead of the tax burden. Notice that the fiscal freedom index is an index which gets smaller as statutory taxes increase. So positive sign of its coefficient is expected.

In addition to these estimations, I replicate the same analysis using a measure of tax burden which I obtain by dividing total tax revenue by GDP subtracted by the total informal sector size. Signs of the coefficients do not change and t-statistics become even larger. Moreover, suspecting that the tax burden might be endogenous with respect to the size of the informal sector, I also run a system estimation using 3SLS which doesn’t show any evidence against the negative correlation.\(^\text{17}\)

\(^{17}\)Results of further econometric analysis are available upon request.
1.2.2 Do the Data Tell More?

The previous subsection presented results indicating a negative relationship between tax rates and the size of the informal sector. The model I present in the next section to account for this phenomenon relates this finding to political frictions, specifically to the varying degree of political turnover in different countries. As briefly discussed in introduction, the model implies that countries in which the political turnover is high, the level of tax burden is low. However, tax revenues are mainly wasted due to corruption which makes the level of productive public investment also low. This leads to a larger informal sector. This subsection provides empirical evidence to support this argument, i.e. investigates political turnover’s role in results of the previous subsection.

Data

Political Turnover:

In addition to the control variables used in the previous subsection, here I include a measure of political turnover\textsuperscript{18} among the independent variables. Specifically, I use two measures of political turnover. One is the probability of reelection index developed by a recent paper by Brender and Drazen (2008) using election data from a large number of countries. Another mea-

\textsuperscript{18}Table 1.3 provides summary statistics of all the variables used here.
Table 1.3: Summary Statistics

<table>
<thead>
<tr>
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<th>Mean</th>
<th>Std. Deviation</th>
<th>Minimum</th>
<th>Maximum</th>
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</thead>
<tbody>
<tr>
<td>Tax Burden (in %)</td>
<td>0.19</td>
<td>0.15</td>
<td>0.03</td>
<td>0.47</td>
</tr>
<tr>
<td>Informal Sector Size (in %)</td>
<td>29</td>
<td>14</td>
<td>8.0</td>
<td>67</td>
</tr>
<tr>
<td>Political Stability Index</td>
<td>9.15</td>
<td>0.98</td>
<td>6.99</td>
<td>11.17</td>
</tr>
<tr>
<td>GDP per-capita(in thousand GK$)</td>
<td>13.65</td>
<td>9.52</td>
<td>1.23</td>
<td>34.76</td>
</tr>
<tr>
<td>Fiscal Freedom Index</td>
<td>70.71</td>
<td>15.27</td>
<td>32.3</td>
<td>99.9</td>
</tr>
<tr>
<td>Corruption Index</td>
<td>3.09</td>
<td>1.18</td>
<td>0.6</td>
<td>6</td>
</tr>
<tr>
<td>Bureaucratic Quality Index</td>
<td>2.17</td>
<td>1.15</td>
<td>1</td>
<td>4</td>
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<tr>
<td>Probability of Reelection</td>
<td>0.37</td>
<td>0.33</td>
<td>0</td>
<td>1</td>
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</tbody>
</table>

These are cross-section summary statistics of the panel averages. All the variables except the probability of reelection consist of 80 countries. For probability of reelection I have data for only 58 countries.

Sure is obtained from ICRG’s political stability index\(^{19}\) which is a composite measure for government unity, legislative strength and popular support.\(^{20}\)

**Estimation and Results**

The estimations here aim to test the following hypothesis: Political frictions play an important role in the composition and the level of public finance. The estimations investigate the role of political stability as the key frictions. The idea is that, if the political stability is higher, in other words the incumbent is more certain that it will stay in the office, it will direct more of the tax revenues for productive public investment and less for wasteful government spending, specifically office rent and corruptive activities. Even though, the overall tax rate increases due to increasing political stability,\(^{19}\) when using the political stability index I also use the level of democracy index from Polity IV database among the control variables.\(^{20}\) Probability of reelection database is available for 58 countries of 80 countries in my informal sector dataset. Also, it is only a cross-section data whereas the political stability index of ICRG is a yearly panel and available for all the 80 countries from 1999 to 2005.
the change in the composition of public spending makes the formal sector more attractive for households.

The hypothesis above predicts that a higher political stability (or probability of reelection) is associated with lower level of corruption, higher level of productive government spending, higher tax burden and also a smaller shadow economy. In this section, I provide some empirical evidence for these predictions.

The results of this section’s analysis are presented in different panels of the tables 1.4 and 1.5. The first column, Pooled 1, reports the cross-section regression results with probability of reelection as a measure of political stability. Other columns use ICRG’s political stability index instead. First, in table 1.4, I use tax burden as the dependent variable and estimate several equations with it. The estimations support the hypothesis, namely the positive relationship between the tax burden and political stability. Next, I estimate the relationship between corruption and political stability. Results support the hypothesized negative relationship. Moreover, political stability also seems to be positively correlated with GDP per-capita. Lastly and most importantly, in table 1.5 I investigate the relationship between the informal sector size and political stability using different equations and estimation techniques. According to the empirical analysis, informal sector size and political stability seem to be negatively correlated. Moreover, once political stability or probability of reelection are controlled for, the correlation
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<tr>
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<tr>
<td>R-squared</td>
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<td>0.21</td>
<td>0.24</td>
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<tr>
<td>Observations</td>
<td>58</td>
<td>80</td>
<td>400</td>
<td>400</td>
<td>240</td>
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<tr>
<td>Hansen J-Test</td>
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<tr>
<td>AR(2) Test</td>
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<tr>
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<td>0.35</td>
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<td>Observations</td>
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<td>80</td>
<td>400</td>
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</table>

All panel regressions include year and country fixed effects. Standard errors are reported coefficient in parentheses.

between the tax burden and the informal sector size, even though negative, deceases to be significant. To close the order of the logic it would be nice
Table 1.5: Informal Sector and Political Turnover

<table>
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<tr>
<th></th>
<th>Pooled 1</th>
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<td>(0.72)</td>
<td>(0.82)</td>
<td>(0.89)</td>
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</tr>
<tr>
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<td>-0.05</td>
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<td>(1.4)</td>
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<td>R-squared</td>
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<tr>
<td>Observations</td>
<td>58</td>
<td>80</td>
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<td>0.18</td>
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</tbody>
</table>

All panel regressions include year and country fixed effects. Standard errors are reported coefficient in parentheses.

to get some results on the relationship between productive public spending and political stability. Unfortunately, since there isn’t any widely accepted way of distinguishing between productive and wasteful public spending in the data, I cannot report any results about this. However, there are some empirical studies supporting the logic of my paper. 21,

1.3 Model

In this section I present the model of the paper. First, I describe the general environment and define the competitive equilibrium. Then, I describe

---

21 Kneller et.al. (1999) distinguish between productive and unproductive expenditures in a government spending database of a subset of OECD countries and conclude that productive government spending is positively associated with income and growth. Fiva and Natvik (2009) come to a similar conclusion using local data from Norway. Also, Mauro (1998) finds evidence that corruption is negatively associated with productive government spending.
the Markovian environment, define a politico-economic (symmetric, differentiable, interior Markov-perfect) equilibrium and characterize it. Next, I provide and discuss the analytical solution in a simplified environment. Lastly, I briefly discuss an extension of the benchmark model.

To study the relationship between taxes and the size of the informal sector, I use a two-sector growth model with public investment. In this economy, there is a unit measure of households and a government.

1.3.1 Households

Households can divide their labor endowment between two sectors: formal and informal. These two sectors produce a single non-storable consumption good. Specifically, a stand-in household maximizes the following discounted utility from consumption\(^{22}\):

\[
\sum_{t=1}^{\infty} \beta^{t-1} U(c_t)
\]

subject to the following budget constraint:

\[
c_t + k_{t+1} - (1 - \delta_k)k_t = r_kk_t(1 - \tau_k) + w_f n_f (1 - \tau_n) + y_i(1 - \delta_i)
\]

In the benchmark model, I assume that leisure is not valued. Since adding leisure involve no significant changes in the main results at the expense of much more notation, I decided not to include the extension in this version of the paper. However, I shortly discuss the implications of relaxing this assumption at the very end of this section.
and the time constraint

\[ n_{ft} + n_{it} = 1 \]

where \( n_{ft} \) is the amount of time the household spends in the formal labor market, and \( n_{it} \) in the informal labor market. Labor and capital income in the formal sector are taxed at rates \( \tau_{kt} \) and \( \tau_{nt} \) respectively. Moreover \( r_t \) and \( w_{ft} \) stand for the rental rate of capital and formal wage rate respectively. \( y_{it}(n_{it}) \) represent the informal sector income. Lastly, \( \delta_k \) is the depreciation rate for private capital. The budget constraint suggests that a household has 3 sources of income: Labor and capital income in the formal sector net of taxes (the first two terms on the right hand side of the budget constraint) and income from the informal sector. Hence, given \( k_1, \{ r_t, w_t, \tau_{kt}, \tau_{nt} \}_{t=1}^{\infty} \) the representative consumer’s problem can be written as:

\[
\max_{c_t, k_{t+1}, n_{ft}} \sum_{t=1}^{\infty} \beta^{t-1} U(c_t)
\]

subject to the following budget constraint

\[
c_t + k_{t+1} - (1 - \delta_k)k_t = r_t k_t (1 - \tau_{kt}) + w_t n_{ft} (1 - \tau_{nt}) + y_{it}(n_{it})
\]

and the non-negativity and time constraints

\[
c_t, k_{t+1}, n_{it}, n_{ft} \geq 0
\]
\[ n_{it} + n_{ft} = 1 \]

Simplifying the notation to save some space, one can obtain the following first-order conditions at an interior solution of the consumer’s problem:

\[
-U_{ct} + \beta U_{ct+1} (1 - \tau_{k_{t+1}}) r_{t+1} = 0
\]
\[
U_{ct} (1 - \tau_{n_{t}}) w_{ft} - U_{ct} w_{it} = 0
\]

where \( w_{it} \) stands for the wage rate in the informal sector.\(^{23}\)

1.3.2 Technology

Technology for each firm in the formal sector is given by

\[ y_{ft} = f_{1}(k_{t}, n_{ft}, G_{t}) \]

\( G_{t} \) stands for the productive public capital.

On the other hand, I assume that each firm in informal sector produces according to the following decreasing returns to scale technology\(^{24}\):

\[ y_{it} = f_{2}(n_{it}) \]

\(^{23}\)Moreover, \( U_{t} \) and \( U_{t+1} \) represent the derivatives of the utility function with respect to \( c_{t} \) and \( c_{t+1} \), respectively.

\(^{24}\)Technically, I assume that \( \frac{\partial f_{2}}{\partial n_{it}} > 0 \) and \( \frac{\partial^{2} f_{2}}{\partial n_{it}^{2}} < 0 \)
Notice that the informal sector uses only labor as an input.\textsuperscript{25}

1.3.3 Government

The source of uncertainty in the economy arises due to the following political structure: There are two political parties, party 1 and party 2, which can be in power at any \( t \geq 0 \). Technically, let the state of incumbency be defined at any period \( t \), as \( z_t \in Z_t = \{1, 2\} \). I further assume that the uncertainty follows a Markov process, i.e. at the end of each period, the incumbent political party stays in the office with an exogenous probability of \( \rho \) or loses the office to the other party with probability \( 1 - \rho \), i.e. \( \Pr(z_{t+1} = i | z_t = j) = \Pi_{ij} = 1 - \rho \) and \( \Pr(z_{t+1} = i | z_t = i) = \Pi_{ii} = \rho \), for \( i, j \in \{1, 2\} \).

In other words, \( \Pi_{ij} \), for \( i, j \in \{1, 2\} \) is defined by the following simple two-state Markov chain:

\[
\Pi_{ij} = \begin{bmatrix}
\rho & 1 - \rho \\
1 - \rho & \rho
\end{bmatrix}
\]

One can interpret \( 1 - \rho \) as the measure of the degree of political turnover.\textsuperscript{26}

I also assume that the incumbent balances the government budget each period. In the budget there are two potential sources of revenue: Labor and

\textsuperscript{25} None of the results of the paper would change if I had allowed the informal sector use a lower share of public and private capital than the formal sector. The current setup however simplifies the environment a lot without affecting the basic results. Notice that, this simplifying assumption is also used in Loayza (1996).

\textsuperscript{26} Alternatively \( \rho \) can be interpreted as the degree of political stability or probability of reelection. I use all these three terms interchangeably throughout the paper.
capital income taxes from the formal economy. The incumbent party also
chooses how much of this revenue to spend for productive public investment
$G_{t+1}$ and for the office rent $S_t$.\footnote{27} Hence, the government budget is given by:

$$r_t K_t \tau_k + w_f t N_f t \tau_n = S_t + G_{t+1} - (1 - \delta_g) G_t$$

where $\delta_g$ is the depreciation rate of public capital, $K_t$ and $N_f t$ are the
aggregate private capital and formal labor, respectively.

I further assume that the objective functions of the two political parties
are symmetric, i.e. the period utility of the incumbent party $i \in \{1, 2\}$ is
given by

$$U(C_t) + U^g(S_t)$$

whereas the period utility of the opposition party is simply $U(C_t)$. Notice
that, under this assumption, it doesn’t matter for households whether party
1 or party 2 is in power at any period $t$, because the policy choice of each
incumbent is symmetric, i.e. the same. Therefore, households’ decision is
independent of the party in power. This makes the decision of households
and the competitive equilibrium environment deterministic.\footnote{28}

\footnote{27} $S_t$ can be interpreted as nonproductive public spending, office rent or embezzlement. This is why this is party specific and can only be benefited from when in office.

\footnote{28} To be precise I could have defined the households’ problem and the technologies as functions of the history of the realization of the uncertainty. However, this would only create an excess of notation, without any need for it.
This form of the government utility generated a non-benevolent government which gets utility from the office rent it acquires from the tax revenue, in addition to private consumption. Lastly, I define the aggregate resource constraint of this economy as:

\[ C_t + K_{t+1} + S_t + G_{t+1} = Y_{ft} + Y_{it} + (1 - \delta_k)K_t + (1 - \delta_g)G_t \]

Here, \( Y_{ft} \) and \( Y_{it} \) stand for aggregate formal and informal output, respectively.

### 1.3.4 Competitive Equilibrium

Now, having described the general environment, I can define the competitive equilibrium of this economy for a given policy.

**Definition 1.3.1** For a given government policy \( \prod = \{\tau_k, \tau_n, S_t, G_{t+1}\}_{t=1}^{\infty} \) and \( k_1, G_1 \), a competitive equilibrium for this economy is an allocation vector for households \( \{c_t, k_{t+1}, n_{ft}, n_{it}\}_{t=1}^{\infty} \) and a price vector \( \{r_t, w_{ft}, w_{it}\}_{t=1}^{\infty} \) such that

1. Given prices and government policy, the allocation vector of households solves the households’ problem.

2. Prices satisfy \( r_t = \frac{\partial Y_t}{\partial K_t} \), \( w_{ft} = \frac{\partial Y_t}{\partial N_{ft}} \), and \( w_{it} = \frac{\partial Y_t}{\partial N_{it}} \)
3. Government budget constraint is satisfied.

4. Aggregate resource constraint holds.

**Characterizing Competitive Equilibrium**

The competitive equilibrium is characterized by the following conditions which hold for all $t \geq 1$

1. $C_t + K_{t+1} - (1 - \delta_k)K_t = r_tK_t(1 - \tau_{k_t}) + w_tN_{ft}(1 - \tau_{n_t}) + Y_{it}(N_{it})$

2. $-U_{ct} + \beta U_{ct+1}(1 - \tau_{k_{t+1}})r_{t+1} = 0$

3. $U_{ct}(1 - \tau_{n_t})w_t - U_{ct}w_{it} = 0$

4. $r_tK_t\tau_{k_t} + w_tN_{ft}\tau_{n_t} = S_t + G_{t+1} - (1 - \delta_g)G_t$

5. $C_t + K_{t+1} + S_t + G_{t+1} = Y_{ft} + Y_{it} + (1 - \delta_k)K_t + (1 - \delta_g)G_t$

6. $\lim_{t \to \infty} \beta^t \lambda_t K_{t+1} = 0$

where $\lambda_t$ is the Lagrangian multiplier associated with the household budget constraint at time $t$. The first equation is simply the aggregate household budget constraint, the second equation is the Euler equation from the households' first-order condition. Similarly, the third equation comes from the households' first-order conditions equating the marginal products net of taxes in the formal and informal sectors. The fourth equation is the aggregate resource constraint and lastly, the last constraint is the transversality condition.
1.3.5 Politico-Economic Equilibrium

Environment

The equilibrium concept employed here is the same as that in Krusell, Quadrini, and Rios-Rull (1996), Krusell and Rios-Rull (1999) and more recently Martin (2009). The key assumption is that the government does not commit to any of its future policy choices. In each period, the government acts first, choosing current period policies. The equilibrium is called to be Markov-perfect since the government’s choices depend only on the value of the current periods state, in this case just the aggregate private and public capital stocks. Additionally, I only consider equilibria where policy depends differentiably\textsuperscript{29} on the private and public capital stock. (i.e. I assume that the policy functions are differentiable with respect to the state variables.) Lastly, after the government has moved, the private sector chooses its current period action.

Definition

Consider the two first-order conditions of the problem of the household and notice that in a Markov-perfect equilibrium, the government follows a set of policy functions that are only functions of public and private capital

\textsuperscript{29}For details of an environment with non-differentiable finite-horizon equilibria, see Krusell, Martin and Rios-Rull (2006)
today. After setting \( \varsigma = (K, G) \) to be the vector of state variables\(^{30}\), let me define \( G' = \Gamma(\varsigma), \tau_k = \Theta_k(\varsigma) \) and \( \tau_n = \Theta_n(\varsigma) \) to be these objects. Households will understand that in equilibrium government follows policy functions \( \Gamma, \Theta_k, \text{ and } \Theta_n \); thus, the first-order conditions of the private sector yield stationary decision rules for private capital tomorrow and labor in the formal sector today\(^{31}\) that only depend on the private and public capital stock today. Calling them \( K(\varsigma) \), and \( N_f(\varsigma) \) respectively, I can write the two household first-order conditions in a more compact form as follows:

\[
\begin{align*}
\eta(\varsigma, \varsigma', K(\varsigma'), \Gamma(\varsigma'), N_f, N_f(\varsigma'), \tau_k, \Theta_k(\varsigma'), \tau_n, \Theta_n(\varsigma')) &= 0 \quad (1.3.1) \\
\varphi(\varsigma, \varsigma', N_f, \tau_n, \tau_k) &= 0 \quad (1.3.2)
\end{align*}
\]

The two equations above characterize household behavior for the current period for any arbitrary policy of the current government given that the government follows \( \Theta_K, \Theta_n \) and \( \Gamma \) and thus implement \( K, \text{ and } N_f \).

Moreover, I can define the following aggregate functions for the office rent and private consumption.

\[
\begin{align*}
S(\varsigma) &= rK\tau_k + wN_f\tau_n - G' + (1 - \delta_g)G \quad (1.3.3) \\
C(\varsigma) &= rK(1 - \tau_k) + wN_f(1 - \tau_n) + Y_i - K' + (1 - \delta_k)K \quad (1.3.4)
\end{align*}
\]

\(^{30}\)Also to save some space I define \( \varsigma' = (K', G') \).

\(^{31}\)Notice that informal sector labor is known once the formal sector labor is calculated.
Now, given the perception that governments follows some policy $\Gamma$, $\Theta_K$, and $\Theta_n$ which in turn induces household and government behavior given by $\mathcal{K}(\varsigma)$, and $\mathcal{N}_f(\varsigma)$, I can write the problem of the current incumbent party as follows:

$$V(\varsigma) = \max_{\{K', G', N_f, \tau_k, \tau_n\}} U(C(\varsigma)) + U^g(S(\varsigma)) + \beta\{\rho V(\varsigma') + (1 - \rho) W(\varsigma')\}$$

subject to the equations (1.3.1), (1.3.2), (1.3.3) and (1.3.4).

Also notice that

$$W(\varsigma) = U(C^*) + \beta\{\rho W(\varsigma^*) + (1 - \rho) V(\varsigma^*)\}$$

is the value function of the current opposition party where $C^*$ and $\varsigma^* = (K^*, G^*)$ are consumption, tomorrow’s private and public capital, respectively, chosen by the incumbent.

I restrict my focus on (differentiable) symmetric Markov-perfect equilibria (SMPE) of the above described game. This leads to the following definition of equilibrium:

**Definition 1.3.2** An interior SMPE is defined by two value functions $W(\varsigma)$ and $V(\varsigma)$ and policy functions, $\mathcal{K}$, $\Gamma$ $\Theta_K$, $\Theta_n$, $\mathcal{N}_f$ such that for all $K \in (0, \bar{K}]$ and for all $G \in (0, \bar{G}]$, where $K^* = \mathcal{K}(K^*, G^*) < \bar{K}$ and $G^* = \Gamma(K^*, G^*) < \bar{G}$.
and given the Markov chain regulating the probability of reelection \( \rho \) the following conditions are satisfied:

1. Given the value functions \( W(\varsigma) \) and \( V(\varsigma) \), policy functions \( \mathcal{K}, \Gamma \Theta \)
   \( \Theta_n, \mathcal{N}_f \) solve the government maximization problem for the variables
   \( K', G', \tau_k, \tau_n, \) and \( \mathcal{N}_f \), respectively.

2. Given the policy functions \( \mathcal{K}, \Gamma \Theta \)
   \( \Theta_n, \mathcal{N}_f \) value functions \( W(\varsigma) \) and \( V(\varsigma) \) satisfy the functional equations defined above.

3. Policy functions are differentiable in both of their arguments.

Characterizing Markov-Perfect Equilibrium

In this subsection, I characterize the interior symmetric differentiable Markov-perfect equilibrium in the general environment defined above. To this end, I state the following characterization theorem:

**Proposition 1.3.3** The interior symmetric differentiable Markov-perfect equilibrium (interior) is a set of smooth functions \( \{\Theta_n, \Theta_k, \mathcal{K}, \Gamma, \mathcal{N}_f\} \), that for all \( K \in (0, \bar{K}] \) and \( G \in (0, \bar{G}] \) satisfy the equations (1.3.1) and (1.3.2), together with the following equations:

\[
\Theta_n = 0
\]
\[-U_c + \lambda F_1 + \beta [\rho U_s[Y_{K'} + 1 - \delta_k]] + \beta (1 - \rho)\]

\[
\left\{ U'_c \left\{ [1 - \gamma \tau_{K'}] Y_{K'} - K'_{K'} + 1 - \delta_k \right\} + \beta \frac{K'_{K'}}{1 - \rho} \frac{U_c' - \lambda F_1'}{\beta} + (1 - 2\rho) U_c'[Y_{K''} + 1 - \delta_k] \right\}
+ \beta \frac{F_{G'}}{1 - \rho} \frac{U_s' - \lambda F_2'}{\beta} + (1 - 2\rho) U_s'[Y_{G''} + 1 - \delta_y] + \lambda F_3' = 0
\]

\[-U_s + \lambda F_2 + \beta [\rho U_s[Y_{G'} + 1 - \delta_g]] + \beta (1 - \rho)\]

\[
\left\{ U'_c \left\{ [1 - \gamma \tau_{K'}] Y_{G'} - K'_{G'} + 1 - \delta_g \right\} + \beta \frac{K'_{G'}}{1 - \rho} \frac{U_c' - \lambda F_1'}{\beta} + (1 - 2\rho) U_c'[Y_{K''} + 1 - \delta_k] \right\}
+ \beta \frac{F_{G'}}{1 - \rho} \frac{U_s' - \lambda F_2'}{\beta} + (1 - 2\rho) U_s'[Y_{G''} + 1 - \delta_y] + \lambda F_4' = 0
\]

**Proof.** See Appendix 4.1.1 □

The first equation above simply states that all the burden of taxation in this environment falls on capital. The other two equations are the generalized euler equations characterizing the Markov-perfect equilibrium. Even though they seem somewhat complicated and the derivation of them are quite difficult, they show two simple things and are very intuitive: For example, the second one shows the trade-off that the incumbent faces by investing one more unit of public capital today. Investing one more unit of public capital \(G_{t+1}\) today directly reduces \(S_t\) by one unit. That is why the second equation starts with the term \(-U_s\). It also distorts the euler equation of the households which is represented by the term \(\lambda F_2\). However, depending on the value of \(\rho\) it brings benefits tomorrow and thereafter. These benefits are represented by the terms after \(\beta\). With probability \(\rho\), the incumbent of today stays as the incumbent tomorrow and continues to
enjoy the office rent, which is represented by the term $\beta(\rho U_s' [Y_{G'} + 1 - \delta_{Y}])$.

On the other hand, the incumbent loses the power with probability $1 - \rho$.

However, even if it loses the power, it can still affect the decisions of the next period’s government. This is because the current incumbent plays as a Stackelberg leader against the next period’s incumbent. This incumbency advantage of the current incumbent is represented by the last three terms in the curly bracket.

In a similar fashion, the first equation illustrates the trade-off the incumbent faces by investing one more unit of private capital today. All the discussion for the second equation above also applies to the first one.

A Simple Finite Period Analysis

Before conducting numerical experiments with the general environment of the infinite horizon economy which is characterized above, here I first discuss the Markov-perfect equilibrium in a much simpler finite-period economy. The finite horizon allows me to get certain crucial analytical results under some specific simplifying assumptions. On the other hand, in the next section I present numerical solutions of the infinite horizon economy without using some of the specific assumptions below.

Now, for this subsection I make the following assumptions on the form of the utility and production functions and the depreciation rates of private and public capital:
Assumption 1 \( U(C_t) = \alpha_c \log(C_t) \) and \( U^g(S_t) = \alpha_s \log(S_t) \), where \( \alpha_c + \alpha_s = 1 \)

Assumption 2 \( Y_{ft} = F_1(K_t, N_{ft}, G_t) = K^\gamma N_{ft}^{1-\gamma}(G_t / K_t)^\gamma \) and \( Y_{it} = F_2(N_{it}) = N_{it}^{1-\gamma} \)

Assumption 3 \( \delta_k = \delta_g = 1 \)

Notice that in this setup the formal sector production function exhibits constant returns to scale both at individual and aggregate levels. Barro and Sala-i Martin (1992) argue that the way that \( G \) enters the formal sector production function with congestion reflects public goods which are rival but not excludable. However, since the informal sector cannot utilize these public goods in this setting, makes them excludable for the informal sector.\(^{32}\)

Assume for now that the economy only lasts for \( T \) periods and \( T = 2 \). Below I consider the symmetric Markov-perfect equilibrium in this environment. By definition, in a Markov-perfect equilibrium, households and the government base their decisions only on the current state variables; in this case, the aggregate private and public capital stock at the beginning of each period.

\(^{32}\)Notice that, using other forms of production functions without congestion, such as \( Y_{ft} = K^\gamma N_{ft}^{1-\gamma} G_t^{1-\gamma-\beta} \) wouldn’t change the results. However, it would make the household’s problem more complicated due to the fact that the production function would be of decreasing returns to scale in individual firm level.
The timing of choices in this setup is as follows: In the first period, the incumbent, after observing \( G_1 \) and \( K_1 \) (which are initially given), chooses \( S_1, G_2, \tau_{n_1}, \) and \( \tau_{k_1} \) subject to the government budget constraint, taking the following as given:

1. Households maximize utility subject to their budget constraints and markets are competitive.

2. The policy implemented by the government in period 2, which is a function of \( K_2 \) and \( G_2 \).

3. The exogenous probability \( \rho \) of keeping the office in period 2.

In the second period the government in office observes \( K_2 \) and \( G_2 \) and chooses \( \tau_{n_2}, \tau_{k_2}, \) and \( S_2 \), taking as given that households maximize utility. I further assume that the incumbent lacks commitment, even if it had been in power in the previous period which implies that the government in period 2 will not internalize how it’s actions affected the decisions made in period 1.

Lastly, using the timing described above, I solve the model by backward induction. The results can be summarized by the following proposition:

**Proposition 1.3.4** Under assumptions 1-3 and for \( \alpha_s \) small enough\(^{33} \text{ sym-}\)

\(^{33}\text{This assumption is needed for an interior solution. Otherwise, if } \alpha_s \text{ is above some threshold value, then the incumbent confiscates the entire private capital stock and this shuts down the economy.}\)
metric Markov-perfect equilibrium allocations of the first period feature.\textsuperscript{34}

1. Tax rate on formal labor income is zero in both periods.

2. Tax burden falls on capital in both periods.

3. As probability of reelection (\(\rho\)) increases, the first-period incumbent invests more in the productive public good and spends less in the office rent.

4. As probability of reelection increases, the increase in the productive public investment is more than decrease for the first-period office rent, i.e. the tax burden in the first period also increases.

5. An increase in the probability of reelection reduces the amount of labor spent in the informal sector for the second period.

6. An increase in the probability of reelection reduces the size of the informal sector in the second period.

**Proof.** See Appendix 4.1.2

Notice that, the results of the two-period model can be somewhat misleading for the desired results of the paper. This arises due to the fact

\[
\frac{\partial \tau_1}{\partial \rho} > 0, \quad \frac{\partial G_2}{\partial \rho} > 0, \quad \frac{\partial S_1}{\partial \rho} < 0, \quad \frac{\partial (G_2 + S_1)}{\partial \rho} > 0, \quad \frac{\partial N_f}{\partial \rho} < 0, \quad \frac{\partial N_i}{\partial \rho} < 0, \quad \frac{\partial Y_f}{\partial \rho} > 0, \quad \frac{\partial Y_i}{\partial \rho} < 0
\]
that the finite period model implicitly assumes that \( T = 2 \) is the end period, where no private and public investment is made anymore. Obviously, such a period does not exist for the infinite horizon economy. Also, for a two-period economy, some of the first-period allocations generally depend on the initially given state variables, namely \( K_1 \) and \( G_1 \). However, the two-period model still provides helpful insights for the understanding of the main mechanism of the model which will still be valid for the results of the infinite horizon economy.

The formal proof of the proposition is provided in the appendix, however below I briefly discuss the intuition of the above stated results.

First result in the above proposition states that the tax rate on formal labor in both periods is equal to zero and the burden of taxation falls on capital. The labor tax in the second period is equal to zero, because the incumbent of the second period is facing a static problem and due to the existence of an informal sector, the tax on formal labor income is distortionary, whereas since the capital of the second period is already invested, the capital income tax is not distortive. Hence, all the burden of taxation falls on capital. However, the tax rate on capital in the second period depends on the value of \( \alpha_s \), and \( \tau_{k2} < 1 \) if only if \( \alpha_s \) is sufficiently low. Otherwise, \( K_2 = 0 \) and the economy shuts down in the second period. That is why an interior solution requires \( \alpha_s \) to be small enough. Now, under this assumption, the economy is at the first-best \( (U_{c2} = U_{s2}) \) in the second period because the
only used tax instrument, the capital tax, is non-distortionary. Therefore, both $S_2$ and $C_2$ are constant fractions of the second period total output. Next, using this result, assumption 2 and equation 2, I can express $S_2$, $C_2$, $N_{f2}$ and $N_{i2}$ as functions of $G_2$ only. More specifically, one can also obtain $N_{f2}$ as an increasing function of $G_2$.

Having all the second period allocations derived as a function $G_2$ only, one can write the problem of the incumbent in the first period. Now of course, $\rho$ plays an important role here, because from the first period’s perspective, whether the first-period incumbent will enjoy office rent in the second period or not, depends on the value of $\rho$. In this sense, $\rho$ increases the weight of the office rent of the second period in the first period incumbent’s utility function. Therefore, as the probability of reelection, i.e. $\rho$, increases, the marginal rate of substitution between current office rent and future office rent and the marginal rate of substitution between current private consumption and future office rent changes in favor of the future office rent. This lets the current incumbent decrease the current office rent and increase the tax rate on current capital to reduce current private consumption. With more tax revenue at hand, the incumbent invests more on the productive public investment. Notice that, the labor tax in the first period is also equal to zero and the burden of taxation fall on capital again. However, the government in the first period additionally faces an inter-temporal distortion created by the capital tax in the second period. This distorts the
margin between private consumption and office rent in the first period.\textsuperscript{35}

For the last two statements, one might be curious to ask what happens to the formal and informal sector labor in the first period. Formal and informal sector labor in the first period depend on first period’s stock of private and public capital, $K_1$ and $G_1$, and the labor tax rate. Since $K_1$ and $G_1$ are exogenously given and $\tau_{n1} = 0$, formal and informal labor in the first period together with formal and informal output are fixed. However, formal and informal labor of the second period are functions of the public capital of the second period which is an increasing function of $\rho$. So, as the probability of reelection increases, so do the formal sector labor and the formal sector output in the second period; which in turn reduce the relative size of the informal sector in the second period.

The two-period economy with the simplifying assumptions can be generalized to a an arbitrary $T$ period economy. Moreover, letting $T \to \infty$, I can state the following result for the equilibrium of the infinite horizon economy as the limit of the above described finite horizon economy:

\textbf{Proposition 1.3.5} For $\alpha_s$ small enough and assuming that the assumptions 1, 2 and 3 hold, there exists an interior Markov-perfect equilibrium of the infinite horizon economy in the above described environment in which the steady state statistics feature:

\textsuperscript{35}Technically $U_{c1} < U_{s1}$.
\[ \tau_n = 0, \tau_k > 0, \frac{\partial \tau_n}{\partial \rho} > 0, \frac{\partial (G/Y)}{\partial \rho} > 0, \frac{\partial (S/Y)}{\partial \rho} < 0, \frac{\partial (G+S)/Y}{\partial \rho} > 0, \frac{\partial (Y_i/Y_f)}{\partial \rho} < 0 \]

where \( Y = Y_i + Y_f \). This proposition is actually an extension of proposition 1.3.4. The proof is discussed in appendix 4.1.2. I should also note that proposition 1.3.5 does not actually give much information in addition to proposition 1.3.4. It simply states the key results of the two-period environment extend to an infinite horizon environment.\(^{36}\) Notice that with both propositions at hand, I have an environment in which both the relative size of the informal sector and the tax burden depend on the exogenous probability of reelection. An increase in this probability also increases the tax burden but reduces the size of the informal sector, exactly as we observe in the data.

**Adding Leisure-Labor Choice**

Even though the model above assumes that households do not value leisure, it can easily be extended to include leisure in the utility function without changing main results, most importantly the one concerning the relationship between the tax burden and the size of the informal sector.

\(^{36}\) Notice that the steady state features a labor tax rate which is equal to zero. However, the tax rate on formal labor can be easily be made positive by making the current capital tax also distortionary. One way of doing this is extending the model with endogenous capital utilization by allowing households to choose the amount of private capital to be utilized in the formal sector production function. This way, without changing the desired result of the negative correlation between the tax burden and the informal sector size one can have both positive capital and labor taxes in the steady state. Since such an extension does not change any of this paper’s results, I refer to Martin (2009) for such an extension.
Since it only brings more notation and longer derivations, I decided not to include this extension in this version of the paper. However, I still can state the main results of the model extended with leisure. However, first the assumptions have to be adjusted to the environment with leisure:

**Assumption 4** \( U(C_t, \ell_t) = \alpha_c \log(C_t) + \alpha_{\ell} \log(L_t) \) and \( U^g(S_t) = \alpha_s \log(S_t) \), where \( \alpha_c + \alpha_{\ell} + \alpha_s = 1 \)

Notice that \( L_t \) stands for aggregate leisure. In this environment, I can state the following theorem:

**Proposition 1.3.6** For \( \alpha_s \) small enough and under assumptions 2, 3, and 4 there exists an interior Markov-perfect equilibrium of the infinite horizon economy in the above described environment in which the steady state statistics feature:

\[
\frac{\partial(G/Y)}{\partial \rho} > 0, \quad \frac{\partial(S/Y)}{\partial \rho} < 0, \quad \frac{\partial(G+S)/Y}{\partial \rho} > 0, \quad \frac{\partial Y_i/Y_i}{\partial \rho} < 0
\]

The proof is simply an extension of proposition 1.3.5 and is briefly discussed in the appendix.\(^{37}\)

\(^{37}\)One setback of the environment with leisure is that the tax on formal labor which was zero in the previous environment becomes negative now. So in equilibrium there is a labor subsidy which turns out to be a decreasing function of \( \rho \). The idea here is that the incumbent uses labor subsidy to correct part of the distortion created by the capital tax. One way of having a positive labor tax is to introduce endogenous capital utilization a la Martin (2009). However, all these complications do not involve any significant changes in the main results, therefore are not included in this text.
1.4 Numerical Analysis

This section conducts a quantitative analysis of the model’s results without the assumption 3, i.e. in this section I relax this assumption to the following:

Assumption 5 \( \delta_k \in [0, 1] \) and \( \delta_g \in [0, 1] \),

Moreover, I keep the assumption 1; however, I also relax the assumption 2 to the following:

Assumption 6 \( Y_{ft} = K^{\gamma} N_{ft}^{1-\gamma} \left( \frac{G_t}{K_t} \right)^\mu \) and \( Y_{it} = F(N_{it}) = N_{it}^\phi \)

Krusell and Smith (2003) show that this class of dynamic policy games may feature both differentiable and non-differentiable Markov-perfect equilibria. However, I restrict my attention only on differentiable Markov-perfect equilibrium and numerically calculate the steady state statistics of this economy. I describe the relevant computational algorithm in appendix 41.3.

1.4.1 Parametrization and Calibration

The parameterization of the baseline economy is standard. The capital share, as standard in the RBC literature is assumed to be equal to \( \gamma = 0.36 \). Moreover, I assume that \( \beta = 0.96, \delta_k = 0.08, \delta_g = 0.1 \). Lastly, I take \( \mu = 0.15 \) from Eicher and Turnovsky (2000).
Now, the only remained parameters are $\alpha_s$ in the utility function and $\phi$ in the informal sector production function. These, I calibrate. What I do in the next subsection is that, once I calibrate these two parameters, I take the probability of reelection data given by Brender and Drazen (2008), feed their series into the model as $\rho$ and then obtain generated series of relevant endogenous variables in the steady state, i.e. tax burden, $\frac{r_t K_t \tau_t}{Y_t}$, the relative size of the informal sector, $\frac{Y_i}{Y_f}$, public capital-output ratio $G/Y$, and lastly office rent-output ratio $S/Y$.

1.4.2 Quantitative Results and Experiments

I calibrate $\alpha_s$ and $\phi$ to match the average size of the tax burden and the informal sector size in my dataset.38 Specifically, I calculate the average probability of reelection in the data, feed this average value into the model as the $\rho$ and then back out the values of the two parameters mentioned above required to match the average size of the tax burden and the informal sector size.

In figures 1.4 to 1.7, using the calibrated values for $\alpha_s$ and $\phi$, I plot certain endogenous variables of interest against various values of $\rho$ in the interval from 0 to 1 to see the mechanism behind the model's crucial result. Moreover, this simulation results also gives me numerical evidence in the general environment with partial depreciation rate analogous to the

38The calibrated values are $\alpha_s = 0.11$ and $\phi = 0.45$
Figure 1.4: Informal Sector vs. Probability of Reelection

Figure 1.5: Tax Burden vs. Probability of Reelection
Figure 1.6: Public Investment vs. Probability of Reelection

Figure 1.7: Office Rent vs. Probability of Reelection
theorems proved in the simplified environment with full depreciation.

As figure 1.4 shows, increasing $\rho$ reduces the size of the informal sector, and as the next figure, figure 1.5 shows, it increases the tax burden. However, the two components of government spending go into different directions. As probability of reelection increases, public capital-output ratio goes up and office rent to output ratio goes down. These two are illustrated in figure 1.6 and 1.7. Next, figure 1.8 compares the model’s performance against the data. The model performs remarkably well in accounting for the observed negative relationship between the tax burden and the size of the informal sector. Moreover, finally in figure 1.9, I compare the simple linear regression lines drawn for the actual data and for the model generated data. The slopes are almost the same and the two lines almost overlap.\textsuperscript{39}

1.5 Conclusion

In this paper I have developed a model to account for the surprising negative relationship between the tax burden and the size of the informal sector. First, I established this relationship in a panel data analysis and showed that the empirical result is robust. Moreover, the empirical analysis hints that the key to understanding this phenomenon might be a a specific political friction, namely political turnover. However, existing models of the

\textsuperscript{39}The distance between the two lines is created intentionally completely for visual purposes.
Figure 1.8: Informal Sector and Tax Burden: Data vs. Model

![Informal Sector and Tax Burden: Data vs. Model](image1)

Figure 1.9: Informal Sector and Tax Burden: Data vs. Model Regressions

![Informal Sector and Tax Burden: Data vs. Model Regressions](image2)
informal sector are not capable of accounting for this relationship. Towards this purpose I developed model of fiscal policy with two sectors where the government lacks commitment and incumbency follows a Markov chain with two political parties which can be in power at any time. Political turnover, with the way I introduce it, crucially affects both the level and the composition of government revenue and spending. The lower the turnover, the lower the unproductive office rent and the higher the productive public spending. Moreover the tax burden increases with political stability. Even though the tax burden is higher, the tax revenue is increasingly used for the productive public good in the formal sector which creates incentives for being formal and thereby reduces the relative size of the informal sector.

The contribution of this paper is threefold: First, it mainly contributes to the literature on informal economy and taxes, informal economy and corruption, and informal economy and productive public goods. Noticing that most work done in these areas are empirical and lack a strong theoretical basis, this paper provides a general equilibrium model and fills in the theoretical gap in the literature with a novel mechanism. Second, to the best of my knowledge, this paper is the first attempt to utilize empirical results of a panel data set among the other empirical papers on the informal sector. Lastly, this paper also contributes to the literature on optimal Markov-perfect fiscal policy by adding an informal sector and a political economy dimension to standard models of this literature.
Chapter 2

A Theory of Economic Development with Endogenous Fertility

2.1 Introduction

The process of industrialization or in broader terms economic development can be categorized into three stages (Galor and Weil (2000)\(^1\), Hansen and Prescott (2002), Clark (2005)): The first stage is usually called the Malthusian stage, where low (or no) population growth goes hand in hand

\(^1\)See Galor and Weil (1999) for a non-technical discussion of this.
with low (if any) growth in output per capita. In the second stage of development, called the post-Malthusian stage, output per capita grows together with population which means that the growth rate of output is higher than those of population. Finally, there is the modern stage\(^2\) where income per capita continues to grow whereas the population growth is low (if any).

Even though there are no well defined time periods for the three stages, the Malthusian stage corresponds to the time period up to the end of 1700’s. Clark (2005) characterizes this stage as one with little education and human capital, low productivity, and high gross reproduction rate but much lower net reproduction rate (due to high mortality), in turn leading to low population growth.

The industrial revolution in England, starting roughly sometime between 1760 – 1840 (Floud and McCloskey (1994)), lead to the second stage which lasted more or less up to the 20th century. The fertility rate did not decrease much in the transition (Clark 2005), but the higher decrease in mortality (or increase in life expectancy), as documented in Floud and McCloskey (1994) or in Nerlove and Raut (2003), lead to an increase in population. However, the growth rate of output was higher than the growth rate of population, so in this stage output per capita increased and living standards improved, contrary to the well-known predictions of the Malthusian growth theory.

\(^2\)Galor and Weil (2000) call these stages Malthusian, post-Malthusian and modern growth regimes, respectively. Hansen and Prescott (2002) talk about stages which are only differentiated by the Malthus and Solow production functions. Clark (2005) names them pre-industrial world, transition period, and modern world, respectively.
The last stage in which population growth rates started to decline began in the middle of the 20th century. According to Clark (2005), the main characteristics of this stage are low fertility, increased level of education and human capital, and high productivity growth. The decline in fertility and continuing increase in output is also documented in Galor and Weil (2000), Hansen and Prescott (2002), Doepke (2004), and Bar and Leukhina (2007).

The main purpose of this paper is to build a unified model of economic growth and demographic change which can explain the characteristics of growth in output and population in transition through the process of economic development as described in Lucas (2002).

The model constructed in this paper is a combination of the Malthusian and Solow growth models with an additional human-capital-intensive production function which allows for spill-over effects. It is a one-good standard general equilibrium growth model with two different technologies which differ in their total factor productivities (TFP) and use of factors. The first one, called **primitive** technology is assumed to employ effective labor, reproducible capital and a fixed amount of land. Effective labor is the product of number of workers, the portion of time devoted to work by each worker and the level of human capital that each worker possesses. Human capital for each worker depends on the education of the worker, determined by his parents and the rate of technological change as in Galor and Weil (2000) and Lagerlof (2006). These assumptions will specify a certain functional form for
human capital accumulation. With the help of this specification, we will be able to obtain a formula for optimal fertility as a function of technological improvement, mortality and education. The second technology, named as the modern production function, does not use land as an input, but uses only effective labor and capital and it also allows for spill-over effects.

I will choose the initial conditions so that in the beginning, the economy is on the Malthusian steady state. This means that only the first technology is used in the production process. Proposition 1 (same as in Hansen and Prescott (2002)) proves that it is always profitable to operate the primitive sector. On the other hand, proposition 2 specifies exactly when the modern sector starts to be operated, because it becomes profitable only when enough human capital is accumulated in the economy.

The numerical exercise, I will present at the end of the paper, reflects the characteristics of the three periods discussed in the beginning. The simulation is done for nine periods corresponding to 300-350 years. Assuming that the model economy starts in the early 18th century, I will track the evolution of the variables of the economy up to the end of the 20th century. The model generates series for output, output per capita, population, and fertility which match the data obtained from Floud and McCloskey (1994) and Madison (2007).

This paper is related to other works in the literature. To explain the

---

3 As Hansen and Prescott (2002) assume, we also assume that each period in the OLG model economy corresponds approximately to 35-40 years.

In this literature Hansen and Prescott (2002), Galor and Weil (2000), Tamura (2002) and Bar and Leukhina (2007) deserve more discussion as they are closely related to our model.

Galor and Weil (2000) present a very general 2-period OLG model with a single production function. There are also many papers such as Lagerlof (2003), Lagerlof (2006) and Weisdorf (2006) which used the Galor-Weil model as their framework. The form of the human capital producing technology they specify is assumed to be a function of education and rate of technological progress. The function which I will use for human capital production in this paper is consistent with their specifications. My paper, even though largely consistent with their’s, extends their paper with important modifications and differences along the lines with Hansen and Prescott (2002). Specifically, the analysis of Galor and Weil does not incorporate the structural transformation from a primitive technology to a modern one which was one of the key observations throughout the industrial revolution. In my paper however, this transformation is related to the human capital
accumulation and this is one of the key mechanisms generating the evolution of population in the model.

Hansen and Prescott (2002) is the main point of departure for our paper. They explain the economic transition through the industrial revolution with two production functions without human capital and endogenous fertility. They simply assume that the population growth rate is a function of growth in consumption, and is simply 1 after some point. My paper adds endogenous fertility decision and parental investment in education of children to their model which allows for use of human capital as an input in the production functions. With this I will be able to create a trade-off between the quality and quantity of children which is one of the key tensions in our model to explain population growth. The framework they use provides a simple method to analyze economic development not only for a single economy but also to compare different economies such as in Ngai (2004).

Tamura (2002) presents a model where human capital accumulation causes the economy to switch from agriculture to industry endogenously. In his model, the agricultural sector shuts down after the switch and the industrial sector becomes active. The focus of his paper is the world economic history starting from as early as 3000 B.C. Different than his model, I look at a shorter period, therefore our model performs better in terms of explaining the short-run fluctuations in the data after the industrial revolution.

More recently, Bar and Leukhina (2007) developed a similar model to
ours with endogenous fertility and two production functions without human capital. The most important factors to explain the industrial revolution, they claim, are young-age mortality and TFP. Therefore, the focus of the model in their paper is more in the change of TFP and mortality rates.

This paper is organized as follows: In the next section I discuss some empirical facts from United Kingdom to motivate our model. Section 3 presents the model economy, defines a competitive equilibrium and solves it. Simulation of the model economy in its transition through the three stages is then presented in Section 4. Finally, I offer concluding remarks in Section 5.

### 2.2 Empirical Motivation

The claim that the economic history can be analyzed in three periods can be easily observed when one looks at historical data. One can see the different characteristics of the three periods by looking at GDP, GDP per capita and population figures. Figure 2.1 below illustrates the behavior of the population of the United Kingdom after 1700. The increase in the level of population in the long-run is obvious. But more important is the slope.

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4The data for population are obtained from Floud and McCloskey (1994) and Maddison (2007). One important notice should be made at this moment for all data used in this section. To be able to make better comparison with the simulation, all empirical data presented here averaged out for 35 year periods from 1716 to 1996, e.g. in the following figure the population level in for 1951 is not the actual population in that year, but is the average of population between 1916 and 1951. One exception is for 1716 where the average is taken from 1701 to 1716.
of this curve, namely, the growth rate of population over time.

Figure 2.2 shows the population growth rate derived from the data in figure 2.1. Even though there are some fluctuations, the trend is that the growth rate jumps from a very low level to a higher level after the start of industrial revolution and then decreases over the long-run almost to its original level. Excluding the fluctuations, and looking at the trend, this picture confirms the demographic transition in the three different stages which we hypothesized in the previous section.

There are various reasons why population statistics follow such patterns. Decomposing the growth rate of population to observe the fertility and mortality rates can be a step towards that purpose. For that purpose, figure 2.3 below\(^5\) documents the evolution of the gross reproduction rate (GRR) and the average life expectancy in England. Gross reproduction rate, which was slightly above 2 before the industrial revolution, jumps to almost 3 in the 1820’s but decreases thereafter up to almost 1 at the end of the 20th century.

In the overlapping-generations (OLG) model economy which I will discuss in the next section, the mortality rate will be the probability that the representative agent born at period \(t\) will die before \(t + 1\), which has no counterpart in the data. Therefore, throughout the simulation, I will as-

\(^5\)For GRR and life expectancy, data up to 1871 are taken from Floud and McCloskey (1994). After 1871, GRR is taken from Office of National Statistics, and life expectancy data from the Human Mortality Website: www.humanmortality.org
Figure 2.1: Population

Figure 2.2: Population Growth
Figure 2.3: Gross Reproduction Rate and Average Life Expectancy

Figure 2.4: GDP and GDP per-capita
sume that the average life expectancy documented in the lower panel of the figure 2.3 has a negative relationship with the mortality rate in our model, even though the form of this relationship is unknown. (A specific functional form will be assumed to capture this relation later in the paper.)

For now, the data shows that the average life expectancy increases uninterruptedly after the industrial revolution. Notice that the increase in GRR (gross reproduction rate) and life expectancy positively affects population growth. But when the GRR starts to decrease in time, the population continues to grow as the life expectancy becomes higher. Towards the end of the 20th century, the growth in the life expectancy ceases and GRR decreases (almost to 1) which accounts for the slowdown in the population growth rate. Figure 2.4 looks at GDP and GDP per capita in the United Kingdom. The increasing trend of both variables after the industrial revolution is obvious. As discussed in the introduction, prior to the industrial revolution, the growth in GDP is balanced by the growth in population, so that the growth in GDP per capita is low (if any). But in the second stage both variables start to grow uninterruptedly. As a summary of these figures, one can conclude that the three stages which are discussed in detail in the previous section are observable from the documented data above. Now I can build a model to explain these observations.

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Data after 1821 is taken from Maddison (2007). Data before that is generated using the average growth rates for 30 year periods given in Floud and McCloskey (1994).
2.3 The Model

2.3.1 Households’ Problem

Overlapping generations live for 2 periods. A young household born in period $t$ has the following utility function:

$$\log c_t + \beta (1 - \xi_t) \log c_{t+1} + \gamma n_t^{1-\epsilon} h_{t+1}(e_{t+1})$$  \hspace{1cm} (2.3.1)

Here $c_t$ is consumption of the young household in period $t$, whereas $c_{t+1}$ is its consumption when old. $\xi_t$ is the probability that the young household does not survive period $t$. Besides its own consumption, the representative household can choose the number of children it is going to have, $n_t$, and the amount of education it should invest for its children, $e_{t+1}$. $\gamma$ and $\epsilon$ are simply parameters which show the level of altruism the household has towards its children.

Human capital evolves according to the following equation:

$$h_{t+1}(e_{t+1}, g_t) = \psi(e_{t+1}, g_t)$$  \hspace{1cm} (2.3.2)

where $g_t$ is the rate of technological progress which will be defined more in detail with technology. We further assume that $\psi$ satisfies $\psi_e > 0 \psi_{ee} < 0 \psi_g < 0 \psi_{gg} > 0 \psi_{eg} > 0$.\textsuperscript{7}

\textsuperscript{7}These assumptions are justified and used both in Galor and Weil (2000) and Lagerlof
At any period $t$, the young agent born at $t$ can spend his income for consumption, $c_t$, buying capital, $k_{t+1}$ or land $l_{t+1}$. He earns rent from his capital and land next period. Notice that the depreciation rate is assumed to be equal to 1. The agent’s labor income at period $t$ depends on the wage rate $w_t$, the level of human capital that the agent possesses $h_t(e_t)$, and the amount of time that he spends working, $z_t$. The more he spends his time for work, the less is the amount of education he can provide for his $n_t$ children. Parameters $a$ and $b$ represent the time cost of raising children. (In the simulation, they will be assumed to be fixed numbers.) The agent does not work at $t + 1$.

Accordingly, the households’ budget and time constraints are given by

\[ c_t + k_{t+1} + p_t l_{t+1} = w_t h_t(e_t) z_t \] (2.3.3)

\[ c_{t+1} = r_{K,t} k_{t+1} + (p_{t+1} + r_{L,t+1}) l_{t+1} \] (2.3.4)

\[ z_t + n_t (a + b e_{t+1}) = \bar{z}, \] (2.3.5)

where $p_t$ stands for the relative price of land.

\[ (2006) \]

\[ ^8\text{Robinson (1997) provides a very detailed survey of this literature.} \]
2.3.2 Technology

The model we present in this paper is an OLG model with 2 different technologies. The *primitive* sector employs land, effective labor and physical capital to produce output. The second sector, called the *modern sector*, does not employ land. The production functions are given by:

\[ Y_P = A_P K^\alpha P H^\beta P L^{1-\alpha P - \beta P} \]  
(2.3.6)

\[ Y_M = A_M \eta(S_t) K^\alpha M H^{1-\alpha M} \]  
(2.3.7)

The variables \( A_i, Y_i, K_i, H_i \) and \( L_i \) refer to TFP, output, physical capital, effective labor, and land in sector \( i \in \{P, M\} \). Remember that \( g_t \) is defined to be the rate of technological progress of the economy. With these two production functions in hand,

\[ g_t = \frac{A_t - A_{t-1}}{A_{t-1}}, \]  
(2.3.8)

where \( A_t \) will simply be a weighted average of \( A_P \) and \( A_M \), i.e.

\[ A_t = \frac{Y_P A_P + Y_M A_M}{Y_t}. \]  
(2.3.9)

Throughout the model, land does not depreciate and is fixed at 1. Since
only the primitive sector employs land, this will imply that \( L_{Pt} = 1 \) for any period \( t \).

Consistent with the names of the production function, the modern sector will be capital intensive and effective-labor intensive compared to the primitive sector. Therefore, throughout the paper it will be the case that \( \alpha_P < \alpha_M \) and \( \theta_P < 1 - \alpha_M \).

The modern sector exhibits spill-over effects which are represented by the function \( \eta(S_t) \), where \( \eta'(S_t) > 0 \), \( \eta''(S_t) < 0 \), and \( S_t = N_t h_t \) is the total level of human capital in the economy. Notice that this specification is not new in the literature.\(^9\)

Since the depreciation rate for physical capital is assumed to be 1, the feasibility constraint of the economy\(^10\) is given by

\[
C^t_t + C^{t-1}_t + K_{t+1} = Y_{Pt} + Y_{Mt}.
\]  (2.3.10)

For simplicity it will be convenient to assume that the same firm operates in each sector alone. Given values for \( A_i, w, r_K, r_L \), and \( S_t \), this firm solves the following maximization problem subject to the production functions

\[
max \ Y_i - wH_i - r_K K_i - r_L L_i \quad i \in \{ P, M \}
\]  (2.3.11)


\(^10\)The implicit simplifying assumption made here is that capital in possession of the young who do not survive to the next period is automatically transferred to those who survive.
2.3.3 Equilibrium and Characterization

Given $N_0, k_0, \text{and } \xi_t$ (and assuming that $L_t = 1$ for all $t$), a competitive equilibrium in this economy is defined to be sequences of household allocation $\{c_t, c_t^{t+1}, k_{t+1}, l_{t+1}, z_t, n_t, e_{t+1}\}$, firm allocations $\{K_{M_t}, K_{P_t}, H_{M_t}, H_{P_t}, Y_{M_t}, Y_{P_t}\}$ and prices $\{p_t, w_t, r_{K,t}, r_{L,t}\}$ such that given prices:

1. Households maximize their utility subject to the budget constraints specified above.

2. The representative firm maximizes its profits subject to the production functions.

3. Market clearing conditions hold. Specifically:

\[ H_{M_t} + H_{P_t} = H_t = z_t h_t N_t \]  \hspace{1cm} (2.3.12)

\[ S_t = h_t N_t \]  \hspace{1cm} (2.3.13)

\[ L_{P_{t+1}} = L_{t+1} = l_{t+1} N_t = 1 \]  \hspace{1cm} (2.3.14)

\[ K_{M_t} + K_{P_t} = K_t = k_t N_{t-1} \]  \hspace{1cm} (2.3.15)

\[ C_t^t + C_t^{t-1} + K_{t+1} = Y_{M_t} + Y_{P_t} \]  \hspace{1cm} (2.3.16)

\[ N_{t+1} = n_t N_t. \]  \hspace{1cm} (2.3.17)
Here are some theorems that are worth to state before solving for the competitive equilibrium:

**Proposition 2.3.1** For any wage rate \( w \) and capital rental rate \( r_K \), the firm finds it profitable to operate in the primitive sector. This implies that \( Y_{P_t} > 0 \) for all \( t \).

The proof of this proposition is in Hansen and Prescott (2002).

**Proposition 2.3.2** Given a wage rate \( w \) and capital rental rate \( r_K \), maximized profit per unit of output in the modern sector is positive if and only if

\[ A_{M_t} > \frac{1}{\eta(S_t)} \left( \frac{r_K}{\alpha_M} \right)^{\alpha M} \left( \frac{w}{1 - \alpha M} \right)^{1 - \alpha M}. \]  

(2.3.18)

The proof of this proposition is relegated to the the appendix.

To make use of these propositions, in some period \( t \) we should first calculate

\[ w_t = A_{P_t} \theta_P K_t^{\alpha_P} H_t^{\beta_P - 1} \]  

(2.3.19)

and

\[ r_{K_t} = A_{P_t} \alpha_P K_t^{\alpha_P - 1} H_t^{\beta_P} \]  

(2.3.20)

If the condition of proposition 2 does not hold under these prices, then these are the equilibrium wage and capital rental rate. If proposition 2 holds, then
these are not equilibrium prices; instead, we should use the following system of equations:

\[
  w_t = A_P \theta_P K_t^{\alpha_P} H_t^{\beta_P} = A_M \eta(S_t)(1 - \alpha_M)K_t^{\alpha_M} H_t^{1-\alpha_M} \quad (2.3.21)
\]

and

\[
  r_{K,t} = A_P \alpha_P K_t^{\alpha_P-1} H_t^{\beta_P} = A_M \eta(S_t)\alpha_M K_t^{\alpha_M-1} H_t^{1-\alpha_M}. \quad (2.3.22)
\]

In each period \( t \), using these equalities and the market clearing conditions, it is straightforward to calculate \( K_t, H_t, K_M, \) and \( H_M \).

Now consider the households’ maximization problem. First notice that from the first-order conditions one directly obtains an expression for \( e_{t+1} \) which directly determines \( h_{t+1} \)

\[
e_{t+1} = c(g_t - a), \quad (2.3.23)
\]

where \( c > 0 \) is a constant in terms some of the parameters of the model.

First-order conditions also yield:

\[
p_{t+1} = p_t r_{K,t+1} - r_{L,t+1}. \quad (2.3.24)
\]
Moreover, the budget constraint implies

\[ N_t(w_t z_t h_t - c_t^f) - p_t = K_{t+1}, \quad (2.3.25) \]

and if we combine the budget constraint and first-order conditions we obtain

\[ c_t^f = \frac{w_t h_t z_t}{1 + \beta(1 - \xi_t)}. \quad (2.3.26) \]

Lastly, from first-order conditions we can derive

\[ n_t^* = \frac{\gamma (1 - \epsilon) h_{t+1} z_t}{(1 + \beta_1(1 - \xi_t))(a + b e_{t+1})}. \quad (2.3.27) \]

Equations (27) and (5) yield a system of 2 equations and 2 unknowns: \( n_t \) and \( z_t \). Given values of the parameters and \( \xi_t \), it is straightforward to solve for both of them. Careful examination of equation (27) reveals that \( n_t \) also depends on the rate of technological progress through \( e_{t+1} \). Everything being equal, this captures the Malthusian idea that technology may limit population growth as in Kremer (1993).

Notice that \( N_t \) is the number of young agents (or workers) at any time \( t \), whereas population at \( t \) is given by this number plus the number of old
agents at time $t$, i.e.

$$\pi_t = N_t + (1 - \xi_{t-1})N_{t-1}. \quad (2.3.28)$$

So the population growth rate from $t$ to $t + 1$ is given by

$$\frac{\pi_{t+1} - \pi_t}{\pi_t} = \frac{N_{t+1} + (1 - \xi_t)N_t - (N_t + (1 - \xi_{t-1})N_{t-1})}{N_t + (1 - \xi_{t-1})N_{t-1}}. \quad (2.3.29)$$

2.4 Simulation

Notice that, given sequences of $\{A_{Mt}, A_{Pt}\}_{t=0}$ initial capital stock, and initial number of young agents, $(K_0, N_0$ respectively), the initial price of land $p_0$ and the mortality $\xi_0$, all equilibrium allocations can easily be calculated. All the initial conditions except $p_0$ can be arbitrarily chosen. To compute, $p_0$ I use a numerical (recursive) shooting algorithm similar to one used in Hansen and Prescott (2002). To calculate $A_t$, notice that $g_t$ (which is one of the determinants of $e_{t+1}$ and hence of $z_t$) depends on $A_t$. But $A_t$ depends on $Y_t$, which cannot be calculated without obtaining the value of $z_t$ and hence $A_t$. To overcome this problem I again use a shooting algorithm which will be described below.

Throughout the simulation, I will assume the following functional form
Table 2.1: Values for Basic Parameters

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Description</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$A_P$</td>
<td>TFP in the primitive sector</td>
<td>1.032</td>
</tr>
<tr>
<td>$A_M$</td>
<td>TFP in the modern sector</td>
<td>1.518</td>
</tr>
<tr>
<td>$\alpha_P$</td>
<td>Capital share in primitive sector</td>
<td>0.1</td>
</tr>
<tr>
<td>$\theta_P$</td>
<td>Labor share in primitive sector</td>
<td>0.6</td>
</tr>
<tr>
<td>$\alpha_M$</td>
<td>Capital share in modern sector</td>
<td>0.3</td>
</tr>
<tr>
<td>$\beta$</td>
<td>discount rate</td>
<td>1</td>
</tr>
<tr>
<td>$\gamma$</td>
<td>altruism</td>
<td>0.675</td>
</tr>
<tr>
<td>$\epsilon$</td>
<td>altruism</td>
<td>0.75</td>
</tr>
<tr>
<td>$a$</td>
<td>fixed cost of each child</td>
<td>0.15</td>
</tr>
<tr>
<td>$b$</td>
<td>educational cost of each child</td>
<td>1</td>
</tr>
</tbody>
</table>

for the human capital accumulation function.

$$h_{t+1}(e_{t+1}, g_t) = \frac{a + be_{t+1} - g_t}{a + be_{t+1} + g_t}. \quad (2.4.1)$$

This form\(^{11}\) obviously satisfies the desired properties discussed in Galor and Weil (2000).

Before the simulation exercise I need to perform some calibrations to find the values of parameters. Most of the chosen parameters are consistent with Hansen-Prescott (2002) and Lagerlof (2006). Additionally, $\epsilon$ and $\gamma$ are calibrated to match the GRR’s and population growth rates in 1716 and 1751.

Table 1 below documents the values chosen for the key parameters.

For the modern sector the form of the spill-over effect is assumed to be given by

\(^{11}\)A similar function is also used by Lagerlof (2006).
where $\nu$ is less than 1. First, notice that this specification of the function satisfies the desired properties stated above. Furthermore, since the initial conditions are chosen such that the modern sector is idle at $t = 0$, this requires that $\nu < 0.41^{12}$. Various values are experimented for its value, and the reported simulation takes it to be equal to 0.2.

Moreover, I need values for $\xi_t$, which is the probability that the household does not survive to the second period. The evolution of the average life expectancy in UK is plotted in figure 3.2. Assuming that each period in our model corresponds to a period of 35 years and the life expectancy in UK is normally distributed with the mean values plotted in figure 3.2, we can calculate $\xi_t$. With this we have now all information to do the simulation. The simulation works as follows:

Since I assume that the economy initially is in the steady state with the primitive production function, $g_0 = g_{-1} = A_P$. Therefore, I also have $e_0$ and $h_0$. Given $\xi_0$, $A_P$, $A_M$, $K_0$, $N_0$, and $p_0$, one can then calculate $e_1$, $h_1$, $n_0$ and $z_0$. With these calculations I can obtain the value of every variable in the model. Now, if proposition 2.3.2 does not hold, then I am done with period 0. Assume that this is done up to period $t$ but now proposition 2.3.2

$^{12}$For all other value of $\nu$ the modern sector is active at $t = 0$
holds in period $t$. Then, one cannot assume that $g_t = A_P$, because $A_t$ will not equal $A_{P_t}$. Instead, it will be equal to a weighted average of $A_{P_t}$ and $A_{M_t}$. To calculate the weights of this average, I use a shooting algorithm and guess the weights of the primitive and modern sectors in total production and calculate all the variables accordingly, including $Y_{M_t}$, and $Y_{P_t}$, and the weights. If my guess of the weights is above or below the calculated weights, I update our guess and recalculate. Using this algorithm, I simulate the model economy for 9 periods from $t = 0$. Each period represents 35 years, as the idea is to simulate the transition of population and output from the beginning of the 18th century up to the third millennium.

Below I present the results of the simulation:

Figure 2.5 presents the evolution of the population and population growth rate in the model economy together with the data. The population starts to grow at an increasing rate after the industrial revolution but then its growth rate declines, as it is the case in the data. One reason why the population increases at an increasing rate is that the mortality rate $\xi_t$ (which is exogenous to the model) decreases, as the life expectancy goes up.

We plot the fertility rate $n_t$ in figure 2.6. Notice that the fertility rate increases first (which is the other reason why the population increases at an increasing rate) but then sharply decreases in the following periods almost below 1.

In figure 2.7, we observe what happens to output and output per capita,
Figure 2.5: Population and Population Growth: Data vs. Model

Figure 2.6: Gross Reproduction Rate: Data vs. Model
Figure 2.7: Output and Output per-capita: Data vs. Model

Figure 2.8: Growth of Technological Progress and Time spent for each Child
respectively. Notice that output slowly increases from period 0 but with a parallel increase in the population, output per capita remains stagnant. With the industrial revolution this situation changes and both variables increase together.

Figure 2.8 shows the average rate of technological progress \((g_t)\) and the fraction of time spent for each child. They follow the same pattern as the latter is an increasing function of the former. Time spent for each child goes from a level of 5.25% up to almost 7.8% of total available time of the parent. This is to explain the increase in the education and human capital
of children.

Lastly, figure 2.9 illustrates the evolution of the shares of the primitive and the modern sectors. The primitive sector never shuts down but becomes very insignificant after the fifth period of the model, whereas the modern sector slowly becomes the dominant sector of the economy.

### 2.5 Concluding remarks

This paper builds a unified model of economic growth which can explain the characteristics of growth in output and population in transition throughout the different stages of economic development. For this purpose the two-sector OLG model in Hansen and Prescott (2002) is extended to allow for fertility, education decisions and human capital accumulation to develop a model consistent with Lucas (2002). The simulation of the model economy captures the main characteristics of the data on population and GDP from the British economy between 1750 and 2000. One comparative advantage of the presented model is its simplicity. Various papers examined the same fact in more complicated ways. Though simple, our model is able to explain various observations of the data.

One extension of the present model can be made by endogenizing the mortality rate $\xi_t$. One way of doing this is assuming that the mortality rate is some decreasing and convex function of output per capita which represents
living standards.

Similar simulations can be performed to explain data from various other European countries, but lack of data might be a serious issue here.
Chapter 3

Not-Quite-Great Depressions of Turkey

3.1 Introduction

Dynamic general equilibrium growth models are widely used in modern economics for studying most macroeconomic phenomena, including economic growth, business cycles, and monetary and fiscal policies. Recently, Cole and Ohanian (1999) and Kehoe and Prescott (2002, 2007) opened the way to use them for analyzing economic depressions as well as less severe downturns. In this paper, we follow the great depressions methodology developed in these papers to study growth performance of Turkey for the period 1968-2004.
The great depression methodology has been so far applied to several economies. Among these contributions, the most notable ones include Hayashi and Prescott (2002) for Japan; Beaudry and Portier (2002) for France; Bergoeing et al. (2002) for Mexico and Chile; Kehoe (2003) for Argentina; Conesa and Kehoe (2003) for Spain; Kehoe and Ruhl (2003) for New Zealand and Switzerland; and Conesa, Kehoe and Ruhl (2007) for Finland. The applied dynamic general equilibrium models used in most of these papers involve aggregate production functions that treat total factor productivity as external to the agents, but not as invariant to the policy. Only few papers, such as Conesa, Kehoe and Ruhl (2007), attempt at endogenizing the TFP, with little success though.

To the best of our knowledge, this is the first paper that follows the great depressions methodology to study the Turkish economy. In this study, we inspect growth trends of the Turkish economy and use growth accounting to evaluate the contributions of total factor productivity (TFP), total hours worked, and capital to the output growth. Then, we conduct experiments on calibrated growth models and compare the variables generated by these models with the actual data.

Throughout our period of analysis (1968-2004), the Turkish economy went through two major periods of stagnation. The first one is the deep recession in the period 1977-1984. Being quite severe and persistent, this downturn almost, but not precisely, satisfies the definition of great depres-
sion suggested by Kehoe and Prescott (2002, 2007). The other period of stagnation, 1991-2001, considerably differs from the former. Within this period, the Turkish economy experienced episodes of considerably high rates of growth. However, these episodes were followed by severe recessions in the years 1994, 1999, and 2001, which contributed the dismal record of 0.65% average growth of real GDP per-capita over the period 1991-2001. Indeed, despite the rapid growth in the period 1984-1990, even the entire period 1976-2001 comes very close to satisfying Kehoe and Prescott’s (2002, 2007) definition of great depression. Since neither period exactly satisfies the conditions for a great depression, as also Conesa, Kehoe and Ruhl (2007) does for the Japanese and the Finnish recessions, we call these periods as “not-quite-great” depressions of Turkey.

Our findings from the growth accounting exercise indicate that TFP is the main determinant in the evolution of the output per-working age person. That is, as TFP grows, output grows as well; and as TFP stagnates, so does the output. The capital-output ratio also contributes positively to the growth of output per working age person from 1968 to 2004. The increase in the capital-output ratio is significant, especially in periods where TFP stagnates; e.g., the periods 1976 - 1984 and 1991 - 2001. As for hours of work, the general trend of hours per working age person is decreasing. Therefore, its contribution to growth in output is negative, except in the period 1991 - 2001.
Our benchmark model, absent of distortionary taxes and capital adjustment costs, closely predicts the evolution of output working age person. However, it does not perform well in predicting the path of capital-output ratio and hours worked per working age person. Even though adding taxes and adjustment costs one at a time improves the results upon the benchmark case, the simulation with both capital adjustment costs and taxes performs best. This suggests that rigidities affecting capital accumulation and distortionary taxes have a crucial role in explaining the evolution of capital and hours worked in the Turkish economy.

The rest of the paper is organized as follows: In the next section, we document the growth performance of the Turkish economy and conduct a growth accounting exercise. In section 3, we present the theoretical framework of our analysis. In the first subsection of this section, we introduce a standard one-sector dynamic general equilibrium growth model as the benchmark model of this paper. In the following subsections, we extend this model by incorporating capital adjustment costs and taxes, both separately and jointly. In section 4, we perform numerical experiments to evaluate the performance of the different specifications of the model to account for the data. Finally, we conclude.
3.2 Evolution of the Turkish Economy

In this section, we will first inspect the evolution of GDP per working age person in Turkey through the lenses of the great depression literature. Following that, we will perform a growth accounting exercise to identify the sources of growth.

3.2.1 Inspecting the GDP data

Figure 1 illustrates the evolution of GDP per working age person in Turkey from 1950 up to 2007 together with different trends. The average growth rate of GDP per capita in this period was approximately 2.75%.

Figure 3.1 also shows that the growth performance of Turkey should be evaluated in at least two subperiods. A visual inspection of the figure reveals that something changes after 1976. The average growth rate of Turkey from 1950 up to 1976 was 3.43%, whereas it was only 1.28% from 1976 up to 2001. This number goes up to 2.1% if one extends the endpoint of the latter interval up to 2007.

Figure 2 compares the actual performance of the economy with trends of 2%, 2.75%, and 3.43% constant growth rates applied after 1976. Again, notice that 3.43% was the average growth rate from 1950 up to 1976 and 2.75% was the average growth rate between 1950 and 2007. We also use the 2% trend growth rate, which is the choice of Kehoe and Prescott (2002,
Figure 3.1: Real GDP per person in Turkey, 1950-2007

Figure 3.2: Real GDP per person in Turkey, 1976-2007
Following the figure 3.2, figure 3.3 plots the detrended GDP per working age person series using these different trends.

The choice of the relevant trend growth rate deserves some discussion because it will determine the depths of recessions and/or depressions in our analysis of the Turkish economy. Kehoe and Prescott (2002, 2007) argue that one should use the 2% percent trend growth rate, which is approximately the average growth rate of USA throughout the 20th century. On the other hand, Cole and Ohanian (1999) use the average growth rate of USA between 1919

2007) for the analysis.

86
and 1997, excluding the depression years and come up with 1.9%. Similarly, Beaudry and Portier (2002) use 2.98% France, which is the average growth rate of GDP per capita in France throughout the 20th century, excluding the depression years between 1930 and 1939. The choice of the relevant trend rate for Turkey will not only determine the depths of the recessions but also whether we can name several periods in Turkey as a great depression or not.

Kehoe and Prescott (2002, 2007) define a great depression as follows: An economy is in a great depression in the time period $T = [T_1, T_2]$, if it satisfies three conditions:

1. There exists some $t \in T$, s.t. $\frac{y_t}{g^{-r_1}Y_{T_1}} - 1 \leq -0.20$

2. There exists some $t \leq T_1 + 10$, s.t. $\frac{y_t}{g^{-r_1}Y_{T_1}} - 1 \leq -0.15$

3. There are no $T_1$ and $T_2$ in $T$, such that $T_2 \geq T_1 + 10$, and $\frac{y_{T_2}}{g^{T_2 - r_1}Y_{T_1}} - 1 \geq 0$

where $y_t = Y_t/N_t^2$ for any $t$, and $g$ is the relevant trend growth rate which is chosen to be equal to 1.02 by Kehoe and Prescott (2002, 2007). As it is understood from the definition these three criteria correspond to the depth, rapidity and sustainability of the depression, respectively.

1 The original version of the paper (Kehoe and Prescott (2002)) only requires the first two of the three conditions here.

2 $y_t$ is originally defined to be GDP per working-age person, however when availability of data is an issue Beaudry and Portier (2002), Perri and Quadrini (2002) and Kydland and Zaragaza (2002) used per-capita variables instead. Alternatively, we also used the GDP per-worker data from Penn-World Tables which actually makes the depressions of Turkey look worse. Results obtained using this data are available upon request.
Given this definition, if we take $g$ to be equal to 1.02, a visual inspection of figure 3 reveals that the period from 1977 to 1984 satisfy the second and the third criteria, but not the first one, because the GDP per working age person does not fall up to 20%, but only to 16% below trend. But, if we take $g$ to be equal to 1.0275 or 1.0343, things change. One can see from figure 3 that all the criteria of the definition are now satisfied in both cases.

One can also suspect whether there are any other periods which might be considered as a great depression. The answer is not quite yes. The only year, where it comes close to satisfy the definition, is in 2001, where the GDP per capita falls to almost 20 % below trend, even with respect to the conservative choice of a trend rate of 2%. But that downturn of the economy was not sustained and the economy started to grow at higher rates after 2002. However, as it is also noted in Imrohoroglu et. al (2010) the period between 1977 and 2001 almost satisfies the above stated definition of a great depression. It goes without saying that it is more important to understand the underlying causes of these downturns of the Turkish economy rather than giving names to them. This is what we do in the following sections.

3.2.2 Growth Accounting

To evaluate the contributions of different factors to the changes in output per working age person, we set up an accounting framework based on the neoclassical growth model.
We use the standard Cobb-Douglas production function, which is of the form:

\[ Y_t = A_t K_t^\alpha H_t^{1-\alpha} \]  

(3.2.1)

where \( Y_t \) is the output at the end of year \( t \), \( K_t \) is the quantity of capital stock, \( H_t \) is the total hours worked, and \( A_t \) is the TFP.

We calculate TFP by the following equation:

\[ A_t = \frac{Y_t}{K_t^\alpha H_t^{1-\alpha}} \]  

(3.2.2)

We, then, compile data on output, total hours worked and investment from national accounts.\(^3\)

To create the capital stock series we simply employ the the perpetual inventory method using the following system of equations:

\[ K_{t+1} = K_t (1 - \delta) + I_t \]  

(3.2.3)

\[ \frac{K_{1950}}{Y_{1950}} = \frac{1}{10} \sum_{t=1951}^{1960} \frac{K_t}{Y_t} \]  

(3.2.4)

Equation (3) is the standard law of motion for capital. Equation (4) is based on the assumption that the capital-output ratio of the initial period

\(^3\)The sources of data are discussed in the appendix.
should match the average capital-output ratio over some reference period. Here, we choose the capital stock so that the capital-output ratio in 1950 matches its average over 1951 - 1960.

Equation (3) and (4) make system of 38 unknowns ($K_{1968}$, $K_{1969}$,....,$K_{2004}$ and $\delta$) and 37 equations. We will use another equation, to make $\delta$ consistent with the average ratio of depreciation to GDP observed in the data over the data period used for calibration purposes. Unfortunately, consumption of fixed capital series for Turkey is only available after 1977. So then we find for Turkey that the ratio of depreciation to GDP over the period 1977 - 2004 is

$$\frac{1}{28} \sum_{t=1977}^{2004} \frac{\delta K_t}{Y_t} = 0.0648 \quad (3.2.5)$$

The three equations above yield now enough information to calibrate $\delta$ and create the capital stock series for the period of interest. The calibrated value for $\delta$ is equal to 4.7 %. To our knowledge there is no study on Turkey which calibrates $\delta$, though there are some empirical studies using different values of it. For example, $\delta$ is assumed to be equal to 4.2% per annum in Altug, Filiztekin and Pamuk (2008) and 5% in Ismihan and Metin-Ozcan (2006).

The production function, when written in per working age person terms, becomes
\( y_t = A_t k_t^\alpha h_t^{1-\alpha} \) \hspace{1cm} (3.2.6)

where lower case letters denote per working age person variables. Taking the natural logarithm of equation (6) and manipulating it a bit yields:

\[
\log(y_t) = \log(h_t) + \frac{\alpha}{1 - \alpha} \log\left(\frac{k_t}{y_t}\right) + \frac{1}{1 - \alpha} \log(A_t) \hspace{1cm} (3.2.7)
\]

Equation (7) allows us to decompose growth in output per capita in three factors\(^4\): Changes in TFP, changes in the capital-output ratio and changed in hours of work per capita. Of course, in an economy which is on a balanced growth path, one would expect that changes in output per person are largely, if not all, explained by changes in TFP.

\[
\log\left(\frac{y_{t+1}}{y_t}\right) = \frac{1}{1 - \alpha} \{\log(A_{t+1}) - \log(A_t)\} + \frac{\alpha}{1 - \alpha} \{\log\left(\frac{k_{t+1}}{y_{t+1}}\right) - \log\left(\frac{k_t}{y_t}\right)\} + \log(h_{t+1}) - \log(h_t) \hspace{1cm} (3.2.8)
\]

The result of this growth accounting exercise is graphically presented in figure 3.4 and the numerical results can be checked in column 3 of table 3.1. Both the table and the figure confirm our premise, that TFP is the main determinant of economic growth in Turkey. Capital-output ratio comes next.

\(^4\)Throughout the growth accounting exercise and the simulations of the model we will assume that \(\alpha = 0.35\). In their empirical paper, Ismihan and Metin-Ozcan (2006) suggest that \(\alpha\) of the Turkish economy lies between 0.35 and 0.50. We use different values in this range to check for sensitivity and report only results with \(\alpha=0.35\).
Moreover, the sign of TFP growth also determines the sign of the growth in output per working age person, except the period 1991 - 2001. In this period, following the capital account liberalization in 1989 and ensuring the full convertibility of the Turkish Lira in 1990, even though TFP is decreasing, the increase in the capital-output ratio makes the average growth rate in per capita output positive. As for hours of work, the general trend of hours per working age person is decreasing. Therefore, its contribution to growth in output per-capita is negative, except in the period 1991 - 2001.
3.3 The Dynamic General Equilibrium Model

In this section, we present the theoretical framework of our analysis. First, we introduce the benchmark model. Next, we extend the model by introducing capital adjustment costs and taxes, each separately. Finally, we discuss the complete model both with capital adjustment costs and taxes.

3.3.1 The Benchmark Model

We use the dynamic general equilibrium model in Conesa, Kehoe and Ruhl (2007) as the base model. The model involves an infinitely-lived representative household and a representative firm, both making decisions in perfectly competitive markets. The household’s instantaneous utility function, \( U \), the firm’s production technology, \( F \), and the sequence of TFP, \( A_t \), are exogenous elements of the model.

Taking the wage rate, \( w_t \), and the rental rate of capital, \( r_t \), for each period \( t = 0, 1, \ldots \) as given, the representative household chooses paths of consumption \( \{C_t\}_{t=0}^{\infty} \), working hours \( \{H_t\}_{t=0}^{\infty} \), and capital \( \{K_{t+1}\}_{t=0}^{\infty} \) to maximize her life-time utility

\[
\sum_{t=0}^{\infty} \beta^t \left[ \gamma \log(C_t) + (1 - \gamma) \log(hN_t - H_t) \right] (3.3.1)
\]
subject to

\[ C_t + K_{t+1} = w_t H_t + (1 + r_t - \delta) K_t, \]  
(3.3.2)

\[ C_t, K_t, I_t \geq 0, \]  
(3.3.3)

\[ 0 \leq H_t \leq \bar{h} N_t, \]  
(3.3.4)

\[ K_0 \text{ given}, \]  
(3.3.5)

where \( I_t = K_{t+1} - (1-\delta)K_t \) is investment; \( \beta, \beta \in (0, 1), \) is the discount factor; \( \gamma, \gamma \in (0, 1), \) is the consumption share; \( \delta, \delta \in (0, 1), \) is the depreciation rate of capital; \( \bar{h} \) is the number of hours available to each person for market work and \( \bar{h} N_t \) is the aggregate number of hours available for work.

Equations (10)-(13) are, respectively, the budget constraint, the non-negativity constraints, the time constraint on hours worked and the constraint on the initial capital.

The production technology is given by the equation (1). Taking the prices \( w_t \) and \( r_t \) as given, the representative firm solves the cost minimization problem. The first order conditions, together with the zero-profit condition due to perfect competition, imply the following optimality conditions:

\[ w_t = (1 - \alpha) A_t K_t^\alpha H_t^{-\alpha}, \]  
(3.3.6)

\[ r_t = \alpha A_t K_t^{\alpha-1} H_t^{1-\alpha}. \]  
(3.3.7)
Finally, the feasibility condition is given by

\[ C_t + K_{t+1} - (1 - \delta)K_t = A_tK_t^\alpha H_t^{1-\alpha} \]  \hfill (3.3.8)

**Definition:** Given the sequences of TFP, \( \{A_t\}_{t=0}^\infty \), and population, \( \{N_t\}_{t=0}^\infty \), and the initial capital stock, \( K_0 \); a *competitive equilibrium* is a sequence of prices, \( \{w_t, r_t\}_{t=0}^\infty \), and allocations, \( \{C_t, H_t, K_{t+1}\}_{t=0}^\infty \), such that

1. Given the prices, allocations solve the household’s problem,
2. Allocations satisfy the firm’s optimality conditions (14)-(15),
3. Allocations satisfy the feasibility condition (16).

Next, we will obtain a system of equations that characterizes the equilibrium of the model. First, we derive the first-order conditions from the households utility maximization problem,

\[ w_t(hN_t - H_t) = \left[ \frac{1 - \gamma}{\gamma} \right] C_t \] \hfill (3.3.9)

\[ \frac{C_{t+1}}{C_t} = \beta(1 - \delta + r_{t+1}). \] \hfill (3.3.10)

Then, we insert the prices from the firm optimality conditions (14) and (15) into the household optimality conditions, (17) and (18). Thus, including the feasibility condition (16), we obtain the following system of equations.
that characterizes the equilibrium:

\[
(1 - \alpha) A_t K_t^\alpha H_t^{-\alpha} (\bar{h} N_t - H_t) = \left[ \frac{1 - \gamma}{\gamma} \right] C_t \tag{3.3.11}
\]

\[
\frac{C_{t+1}}{C_t} = \beta (1 - \delta + \alpha A_{t+1} K_{t+1}^{\alpha-1} H_{t+1}^{1-\alpha}) \tag{3.3.12}
\]

\[
C_t + K_{t+1} - (1 - \delta) K_t = A_t K_t^\alpha H_t^{1-\alpha}. \tag{3.3.13}
\]

Given the initial condition \(K_0\), an equilibrium of this model satisfies this system of equations and the following transversality condition:

\[
\lim_{t \to \infty} \beta^t \gamma K_{t+1} C_t = 0. \tag{3.3.14}
\]

In section 4, we will use the equations (19)-(21) to carry out our numerical simulations.

### 3.3.2 Adding adjustment costs to capital accumulation

In this section, we introduce a simple friction into capital accumulation process upon the benchmark model. As in Lucas and Prescott (1971) and Kehoe (2003), we assume there are constant returns to scale adjustment costs to capital stock:

\[
K_{t+1} = (1 - \delta) K_t + \phi (I_t / K_t) K_t \tag{3.3.15}
\]
where
\[ \phi(I_t/K_t) = [\delta^{1-\eta}(I_t/K_t)^\eta + (\eta - 1)\delta]/\eta. \] (3.3.16)

Notice that the case where \( \eta = 1 \) corresponds to the base model with no adjustment costs. Following Kehoe (2003), we will assume \( \eta = 0.9 \) throughout the analysis.

Clearly, this extension only changes the resource constraint of the previous subsection and everything else remains unchanged.

### 3.3.3 Adding taxes

In this section, we follow Conesa, Kehoe and Ruhl (2007) and introduce distortionary taxes into the benchmark model. We assume the government levies proportional taxes on consumption, labor income and capital income and uses the proceed to finance transfers. The household budget constraint (10) in the base model is replaced by

\[ (1 + \tau_C t)C_t + K_{t+1} = (1 - \tau_H t)w_t H_t + (1 + (1 - \tau_K t)(r_t - \delta))K_t + T_t. \] (3.3.17)

where \( \tau_C t \) is the tax rate on consumption, \( \tau_H t \) is the tax rate on labor income, \( \tau_K t \) is the tax rate on capital income, and \( T_t \) is a lump-sum transfer.

Again, the household maximizes her life-time utility function subject to
the budget constraint, the non-negativity constraints, the time constraint, and the initial condition for capital stock, $K_0$.

The firm’s problem is the same as the base problem. Thus, the firm optimality conditions (14) and (15) in the base model are valid in this specification, as well. Since tax revenues are lump-sum rebated back to consumers, the resource constraint is still given by (16).

Finally, the government budget constraints is given by

$$T_t = \tau_{C_t}C_t + \tau_{K_t}(r_t - \delta)K_t + \tau_{H_t}w_tH_t \tag{3.3.18}$$

**Definition:** Given the sequences of TFP, $\{A_t\}_{t=0}^\infty$, population, $\{N_t\}_{t=0}^\infty$, tax policies $\{\tau_{C_t}, \tau_{K_t}, \tau_{H_t}\}_{t=0}^\infty$, and the initial capital stock, $K_0$; a **tax distorted competitive equilibrium** is a sequence of prices, $\{w_t, r_t\}_{t=0}^\infty$, allocations, $\{C_t, H_t, K_{t+1}\}_{t=0}^\infty$, and transfers $\{T_t\}_{t=0}^\infty$ such that

1. Given the prices, allocations solve the household’s problem,
2. Allocations satisfy the firm’s optimality conditions (14) and (15),
3. Allocations, tax policies and transfers satisfy the government budget constraint (26),
4. Allocations satisfy the feasibility condition (16).
3.3.4 Complete Model

Our complete model uses both capital adjustment costs and distortionary taxes. Since we have already defined the equilibrium with and without taxes above, we omit the definition for this case. We refer the reader to Conesa, Kehoe, and Ruhl (2007) for a detailed discussion on solving models of this type.5

3.4 Numerical Experiments

In this section, we first show how we calibrate the remaining parameters of the model, $\beta$ and $\gamma$ and then discuss the simulations of different versions of the model. Lastly, we compare those with the actual data.

3.4.1 Calibration

The calibration procedure is explained in more detail in Conesa, Kehoe and Ruhl (2007). The idea is that as we defined in the previous section, the model features a stand-in household that chooses paths of leisure, investment and consumption to maximize his/her utility. The paths of population and TFP are exogenously given, and the household has perfect foresight over their values. We start the model at date 0, i.e. $T = 1968$ and let time run out to infinity.

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5 Accompanying documentation can also be accessed online at www.greatdepressionsbook.com.
Next, $\beta$, and $\gamma$ are calibrated. In the benchmark model this is done using,

$$
\beta = \frac{C_{t+1}}{C_t(1 - \delta + \alpha Y_t / K_{t+1})} \quad (3.4.1)
$$

$$
\gamma = \frac{C_t H_t}{Y_t (hN_t - H_t)(1 - \alpha) + C_t H_t} \quad (3.4.2)
$$

In the extended versions of the model these equations are replaced by their counterparts.

Moreover, the TFP, which is exogenously given to the stand-in household is calculated using the growth accounting equation derived above.

For the cases with taxes $\beta$ and $\gamma$ are calibrated using,

$$
\beta = \frac{(1 + \tau C_{t+1})C_{t+1}}{C_t(1 + \tau C_t)} \frac{1}{1 + (1 - \tau k_{t+1})(r_{t+1} - \delta)} \quad (3.4.3)
$$

$$
\gamma = \frac{(1 + \tau C_t)C_t H_t}{(1 - \tau_t)Y_t (hN_t - H_t)(1 - \alpha) + C_t H_t} \quad (3.4.4)
$$

Also the TFP is calculated using

$$
A_t = \frac{C_t + I_t}{K_t^{1-\alpha} H_t^\alpha} \quad (3.4.5)
$$

where $C_t + I_t$ is the real GDP at factor prices in the data. However, the
contribution of TFP to growth is reported using

$$\hat{A}_t = \frac{\hat{Y}_t}{K_t^{1-\alpha}H_t^\alpha}$$

(3.4.6)

where

$$\hat{Y}_t = (1 + \tau_C_t)C_t + I_t$$

(3.4.7)

is the real GDP at market prices of the base year $\bar{T} = 2000$

Also notice that, the exogenous sequence working age population is the
Table 3.1: Data and the model without adjustment costs

<table>
<thead>
<tr>
<th>Period</th>
<th>Data</th>
<th>Base Case</th>
<th>Model: Tax</th>
<th>Model: Myopic</th>
</tr>
</thead>
<tbody>
<tr>
<td>1968-2007</td>
<td></td>
<td></td>
<td></td>
<td></td>
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<tr>
<td>change in Y/N</td>
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<td>-0.41</td>
<td>2.43</td>
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one measured from the data in the growth accounting exercise. Following Conesa, Kehoe and Ruhl (2007), we assign a value of $\bar{h} = 100$ for an individual’s time endowment of hours available for market work per week.

The information above is enough to simulate the benchmark model without taxes. For the model with taxes, see the data appendix for calculation of the tax rates.
Table 3.2: Data and the model with adjustment costs

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<td>-0.41</td>
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3.4.2 Simulation Results

Figures 3.5 to 3.16 compare the models predictions against the data. Moreover, last three columns of tables 3.1 and 3.2, perform the growth accounting exercise to the series generated by different versions of the model. In total, we run 6 simulations. Three of them ignore capital adjustment costs. The results of these simulations are reported in table 1 and figures 3.5, 3.6, 3.7, 3.11, 3.12, and 3.13. The remaining three simulations assume that there are capital adjust-
Figure 3.6: Capital/output ratio in Turkey (1976-1984): Data and model simulations

Figure 3.7: Hours worked per person in Turkey (1976-1984): Data and model simulations
Figure 3.8: Detrended real GDP per person in Turkey (1976-1984): Data and model simulations (with adjustment costs)

ment costs. The results of these simulations are reported in table 3.2 and figures 3.8, 3.9, 3.10, 3.14, 3.15, and 3.16. In each of these 2 categories (without and with adjustment costs) of simulations, we run the model first without any taxes, then with constant taxes, denoted by tax1 and lastly with actual taxes, denoted by tax2. 6

In figures 3.5 to 3.10 we only focus on a specific time period, namely the depression years of 1976 to 1984 and compare our models’ performances

6The calculation of the tax rates for the Turkish economy was a daunting task and needs a discussion more than the scope of this subsection. Therefore, we relegate this discussion to the appendix.
against the data. First observation we make from the figures is that the model with constant taxes (with or without capital adjustment costs) improves very little upon the benchmark case. On the other hand, the model with variable taxes (tax 2) is quite successful in predicting the evolution of GDP per working age person, capital-output ratio and hours per working age person between 1976 and 1984.

Also, it is also evident from these figures and from a visual comparison of last columns of tables 3.1 and 3.2 (by comparing the last column of table 3.2 with the data, which is the third column of table 2) that adding capital adjustment costs improves the model’s performance. All these suggest that both rigidities affecting capital accumulation and government policies using distortionary taxes have a crucial role in accounting for the depression years of 1976 to 1984.

Next, in figures 3.11 to 3.16 we look at the general time frame from 1968 to 2004 and compare the model against the data in this period. As both the these figures and the second row in table 3.2 indicate, again the model both with adjustment costs and variable taxes performs the best compared to the alternatives. As the comparison of the third column of table 1 with the third column for the period 1968 and 2004 indicates our benchmark model without any frictions and taxes accounts for 86% of the observed change in GDP per-working age person from 1968 to 2004 and once we extend the model with taxes and capital adjustment costs the comparison of the last
column of table 3.2 with the third column indicates that our extended model accounts for 60% of the observed reduction in hours worked per-working age person and 35% of the change in capital-output ratio from 1968 to 2004.

Also, within the sub-periods we investigate, the only period where none of the models perform well is the period between 1991 and 2001. Considering the high degree of turbulence of the Turkish economy and high degree of political turnover in this period, this shouldn’t be a surprising result.

3.5 Conclusion

In this paper, we use growth accounting and a standard dynamic general equilibrium model to study the growth performance of Turkey between 1968 and 2004. Using the well established great depressions methodology, we find that the primary source of output growth in Turkey was growth in total factor productivity, rather than growth in labor and capital inputs. Among the various specifications of dynamic general equilibrium models employed, the one with capital adjustment costs and variable taxes comes closest to account for the data. This suggests that rigidities affecting capital accumulation and distortionary taxes have a crucial role in explaining the evolution of the Turkish economy. The result also provides evidence that models based on the evolution of TFP alone are generally inadequate for understanding economic growth and recessions. Indeed, our paper highlights
Figure 3.9: Capital/output ratio in Turkey (1976-1984): Data and model simulations (with adjustment costs)

Figure 3.10: Hours worked per person in Turkey (1976-1984): Data and model simulations (with adjustment costs)
Figure 3.11: Detrended real GDP per person in Turkey: Data and model simulations

Figure 3.12: Capital/output ratio in Turkey: Data and model simulations
Figure 3.13: Hours worked per person in Turkey: Data and model simulations

Figure 3.14: Detrended real GDP per person in Turkey: Data and model simulations (with adjustment costs)
Figure 3.15: Capital/output ratio in Turkey: Data and model simulations (with adjustment costs)

Figure 3.16: Hours worked per person in Turkey: Data and model simulations (with adjustment costs)
the importance of recognizing the role of tax policies and rigidities in the capital accumulation process. We believe that those are fertile areas for further research on the Turkish economy, or actually any other developing economy.
Chapter 4

Appendix

4.1 Appendix A: Appendix to Chapter 1

4.1.1 Proof of Proposition 1.3.3

First-order conditions of the maximization problem (specified by the functional equation 5 subject to the constraints 1, 2, 3, and 4 with respect to $K', G', N_f, \tau_k, \tau_n$ are as follows, respectively:
\[ U_c C_K' + \beta (\rho V_K' + (1 - \rho) W_K') + \lambda F_1 + \mu \varphi_K' = 0 \quad (4.1.1) \]
\[ U_c C_G' + \beta (\rho V_G' + (1 - \rho) W_G') + \lambda F_2 + \mu \varphi_G' = 0 \quad (4.1.2) \]
\[ U_c C_{N_f} + U_s S_{N_f} + \lambda \eta_{N_f} + \mu \varphi_{N_f} = 0 \quad (4.1.3) \]
\[ U_c C_{\tau_k} + U_s S_{\tau_k} + \lambda \eta_{\tau_k} + \mu \varphi_{\tau_k} = 0 \quad (4.1.4) \]
\[ U_c C_{\tau_n} + U_s S_{\tau_n} + \lambda \eta_{\tau_n} + \mu \varphi_{\tau_n} = 0 \quad (4.1.5) \]

Notice that \( \lambda \) and \( \mu \) are the Lagrangian multipliers on the constraints \( \eta \) and \( \varphi \), and \( F_1 \) and \( F_2 \) are defined as follows:

\[ F_1 = \eta_{K'} + \eta_{K''} K_K' + \eta_{G''} \Gamma_G' + \eta_{\tau_k} \beta_{kK}' + \eta_{\tau_n} \beta_{nK}' + \eta_{\tau_k} \theta_{kK}' + \eta_{\tau_n} \theta_{nK}' \]
\[ F_2 = \eta_{G'} + \eta_{G''} \Gamma_G' + \eta_{\tau_k} \beta_{kG}' + \eta_{\tau_n} \beta_{nG}' + \eta_{\tau_k} \theta_{kG}' + \eta_{\tau_n} \theta_{nG}' \]

Since \( C_{\tau_k} = -S_{\tau_k} \), \( C_{\tau_n} = -S_{\tau_n} \), \( \eta_{\tau_k} = -\lambda U_c C_{\tau_k} \) and \( \eta_{\tau_n} = -\lambda U_c C_{\tau_n} \), first-order conditions with respect to \( \tau_n \) and \( \tau_k \) can be rewritten as:

\[ -U_c S_{\tau_k} + U_s S_{\tau_k} + \lambda U_c S_{\tau_k} + \mu \varphi_{\tau_k} = 0 \quad (4.1.6) \]
\[ -U_c S_{\tau_n} + U_s S_{\tau_n} + \lambda U_c S_{\tau_n} + \mu \varphi_{\tau_n} = 0 \quad (4.1.7) \]

Notice that \( \varphi_{\tau_k} = 0 \), whereas \( \varphi_{\tau_n} = -U_c \omega_f \). This implies that \( \mu = 0 \) as
long as \( N_f > 0 \). Exploiting this result, \( \lambda \) can be obtained from the first-order condition \( \tau_n \) or \( \tau_k \) as \( \lambda = -\frac{U_s - U_c}{U_{cc}} \).

With this result, and using \( S_{N_f} = \tau_n w_f \), \( C_{N_f} = (w_f(1 - \tau_n) - w_i) \) the first-order condition with respect to \( N_f \) becomes now:

\[
U_c(w_f(1 - \tau_n) - w_i) + U_s\tau_n w_f + \lambda U_{cc}(w_f(1 - \tau_n) - w_i) = 0
\]

Since \( w_f(1 - \tau_n) = w_i \) this implies that \( \tau_n = 0 \), as long as \( N_f > 0 \), so all the tax burden falls on capital every period.

Now, I turn my attention to the first-order conditions with respect to \( K' \) and \( G' \). It can be easily shown that the envelope condition holds for \( V \) but not to \( W \).\(^1\) Hence, after some work I obtain:

\[
\begin{align*}
V_G &= U_s[Y'_G + 1 - \delta_g] \\
V_K &= U_c[Y'_K + 1 - \delta_k]
\end{align*}
\]

Now, forwarding these for one period I get expressions for \( V'_{G'} \) and \( V'_{K'} \). Since I cannot apply the envelope theorem to \( W \), I derive \( W'_{K'} \) and \( W'_{G'} \) by the following operations: First, to get \( W'_{K'} \), I derive \( W \) with respect to \( K \).

\(^1\)This is because the functional equation defining \( W \) is not a maximization problem.
\[ W_K = U_C \{ [1 - \gamma \tau_k] Y^f_K - \mathcal{K}_K + 1 - \delta_k \} + \beta \mathcal{K}_K \{ \rho W'_{K'} + (1 - \rho) V'_{K'} \} \]
\[ + \beta \Gamma_K \{ \rho W'_{G'} + (1 - \rho) V'_{G'} \} + \lambda F_3 \]

where \( F_3 \) the derivative of \( \eta \) with respect to \( K \), i.e. \( F_3 = -U_c C_K + \mathcal{K}_K F_1 + \Gamma K F_2 \)

To find expressions for \( W'_{K'} \) and \( W'_{G'} \), I solve for these from the first-order conditions as

\[ W'_{K'} = \frac{1}{1 - \rho} \left[ \frac{U_c - \lambda F_1}{\beta} - \rho V'_{K'} \right] \]
\[ W'_{G'} = \frac{1}{1 - \rho} \left[ \frac{U_s - \lambda F_2}{\beta} - \rho V'_{G'} \right] \]

Plugging these back into the expression for \( W_K \):

\[ W_K = U_C \{ [1 - \gamma \tau_k] Y^f_K - \mathcal{K}_K + 1 - \delta_k \} + \beta \mathcal{K}_K \left[ \frac{U_c - \lambda F_1}{\beta} + (1 - 2\rho) V'_{K'} \right] \]
\[ + \beta \Gamma_K \left[ \frac{U_s - \lambda F_2}{\beta} + (1 - 2\rho) V'_{G'} \right] + \lambda F_3 \]

Inserting \( V'_{K'} \) and \( V'_{G'} \), into this expression and forwarding one period yields
\[
W'_{K'} = U'_c \{ [1 - \gamma \tau_k'] Y_{K'}' - K'_c + 1 - \delta_k \} + \beta \frac{K'_c}{1 - \rho} \left[ \rho \frac{U'_c - \lambda F'_1}{\beta} + (1 - 2 \rho) U'_c'[Y_{K'}'' + 1 - \delta_k] \right] \\
+ \beta \frac{G'_c}{1 - \rho} \left[ \rho \frac{U'_s - \lambda F'_2}{\beta} + (1 - 2 \rho) U'_s'[Y_{G'}'' + 1 - \delta_g] \right] + \lambda' F'_3
\]

Plugging everything back into the first-order condition with respect to \( K' \) and simplifying yields:

\[
-U_c + \lambda F_1 + \beta \rho U'_c[Y_{K'}' + 1 - \delta_k] + \beta (1 - \rho) \left\{ U'_c \{ [1 - \gamma \tau_k'] Y_{K'}' - K'_c + 1 - \delta_k \} + \beta \frac{K'_c}{1 - \rho} \left[ \rho \frac{U'_c - \lambda F'_1}{\beta} + (1 - 2 \rho) U'_c'[Y_{K'}'' + 1 - \delta_k] \right] \\
+ \beta \frac{G'_c}{1 - \rho} \left[ \rho \frac{U'_s - \lambda F'_2}{\beta} + (1 - 2 \rho) U'_s'[Y_{G'}'' + 1 - \delta_g] \right] + \lambda' F'_3 \right\} = 0
\]

One can get the equation with \( G' \) exactly in the same way

\[
-U_s + \lambda F_2 + \beta \rho U'_s[Y_{G'}' + 1 - \delta_g] + \beta (1 - \rho) \left\{ U'_c \{ [1 - \gamma \tau_k] Y_{G'}' - K'_c + 1 - \delta_k \} + \beta \frac{K'_c}{1 - \rho} \left[ \rho \frac{U'_c - \lambda F'_1}{\beta} + (1 - 2 \rho) U'_c'[Y_{K'}'' + 1 - \delta_k] \right] \\
+ \beta \frac{G'_c}{1 - \rho} \left[ \rho \frac{U'_s - \lambda F'_2}{\beta} + (1 - 2 \rho) U'_s'[Y_{G'}'' + 1 - \delta_g] \right] + \lambda' F'_4 \right\} = 0
\]

where \( F'_4 \) is defined analogous to \( F'_3 \).
4.1.2 Proofs of Propositions 1.3.4 and 1.3.5

Consider the two-period version of the model. I will solve the model by backward induction starting from the second period:

Since the second period is the final period, households do not invest in private capital and consume all of their income. Similarly, the government does not invest in the public capital either. Therefore, from the budget constraint we can write \( C_2 = Y_{f2} + Y_{i2} - S_2 \). Moreover, the labor and capital taxes can be obtained as functions of \( N_{f2} \) and \( S_2 \) only using \( (1 - \tau_{n2})w_{f2} = w_{i2} \) and \( \tau_{k2} = \frac{S_{2}}{\gamma_{f2}} - \tau_{n2} w_{f2} \). Now, for any given \( G_2 \) and \( K_2 \), the problem of the incumbent in period 2 can be written as, choosing \( S_2 \) and \( N_{f2} \) to maximize

\[
U(Y_{f2} + Y_{i2} - S_2) + U^g(S_2)
\]

Under assumption 1, first-order conditions with respect to \( S_2 \) and \( N_{f2} \) respectively are:

\[
U_{c2} = U_{x2}
\]

\[
U_{c2}(w_{f2} - w_{i2}) = 0
\]

Comparing the second equation with equation 2 in the paper, I obtain \( \tau_{n2} = 0 \), i.e. all the tax burden falls on \( K_2 \). Now, the first equation, together with assumption 1 implies that consumption and office rent in the second period
are constant fractions of total output, i.e. \( C_2 = \alpha_c Y_2 \) and \( S_2 = \alpha_s Y_2 \).

Moreover, from the second first-order condition above and assumption 2, I obtain \( \frac{N_2}{Y_2} = G_2^{-1} \). Hence all the second period allocations can be defined as a function of \( G_2 \) only.\(^2\)

Next, using the Euler equation I obtain \( K_2 = m_1 (Y_1 - G_2 - S_1) \), where \( m_1 \) is an increasing function of \( G_2 \).\(^3\) By the resource constraint of period 1, it follows that \( C_1 = (1 - m_1)(Y_1 - G_2 - S_1) \). Next, I consider the maximization problem of the first period incumbent: Given some initial \( K_1, G_1 \), and the probability of reelection \( \rho \), the first period incumbent chooses \( S_1, G_2, \) and \( N_{f1} \) to maximize

\[
U(C_1) + U^g(S_1) + \beta \{ \rho[U(C_1) + U^g(S_2)] + (1 - \rho)U(C_2) \}
\]

or equivalently

\[
U(C_1) + U^g(S_1) + \beta U(C_2) + \beta \rho U^g(S_2)
\]

subject to the following constraints:

\(^2\)Notice that simplifying assumption 2 allows to define all the variables with respect to \( G_2 \) only. In a more general environment, everything should be a function \( K_2 \) and \( G_2 \).

\(^3\)In this specific example \( m_1 = \left\{ 1 + \frac{\alpha_c}{\beta (\gamma G_2 + 1 - \alpha_s)} \right\}^{-1} \).
\[ C_1 = (1 - m_1)(Y_1 - G_2 - S_1) \]
\[ K_2 = m_1(Y_1 - G_2 - S_1) \]
\[ C_2 = \alpha_c Y_2 \]
\[ S_2 = \alpha_s Y_2 \]

So this objective function clearly shows the effect of the \( \rho \). Increasing \( \rho \) affects the marginal rate of substitution between tomorrow’s office rent and current office rent, current private consumption and tomorrow’s private consumption. Notice that \( C_2 \) and \( S_2 \) are functions of \( G_2 \) only. Now, combining the first order conditions with respect to \( S_1 \) and \( N_{f1} \) yields:

\[ U_{S_1}^g (w_{f1} - w_{i1}) = 0 \]

This implies that \( \tau_{n1} = 0 \). Hence, all the burden of taxation falls again on capital. However, the incumbent of the first-period cannot avoid the distortion created by the second period capital tax. This distorts the margin between the private consumption and office rent in the first period. (i.e. \( U_{C1} \neq U_{S1}^g \)) Specifically, the first-order condition with respect to \( S_1 \) implies:

\[ U_{S1}^g = (1 - m_1)U_{C1} \]
This shows that a higher $G_2$ makes $S_1$ more expensive.

With assumption 1, on the form of the utility functions one can also obtain

$$S_1 = \alpha_s (Y_1 - G_2)$$

This equation shows that, given $K_1, G_1$ which since $\tau_{n1} = 0$ directly determine $N_{f1}$ and $N_{i1}$, an increase in $G_2$ implies a reduction in $S_1$. Moreover, the reduction $S_1$ is less than the increase in $G_2$, because $\alpha_s < 1$.

Lastly, the first-order condition with respect to $G_2$ allows us express $G_2$ as a function of initially given $K_1, G_1$, and all the parameters, including $\rho$. Specifically,

$$(\alpha_c + \rho \alpha_s) \gamma = [G_2 (\gamma - \alpha_s) - \alpha_s] f(G_2)$$

where $f(G_2)$ is an increasing function\(^4\) of $G_2$, provided that $\alpha_s < \gamma$. So as one can see from the above equation, increasing $\rho$ increases $G_2$ and hence by the equation defining $S_1$ reduces $S_1$. Moreover, since $\alpha_s < 1$, the increase in $G_2$ if more then the reduction in $S_1$ causing the tax burden of the first period to increase. Since the capital tax is the only tax instrument used by the government, this means that $\tau_{k1}$ increases due to an increase in $\rho$.

Now, having proved the proposition 3.4, one can easily generalize the

\(^4\)Specifically $f(G_2) = \left\{ \frac{1-m_1}{m_1(1-\alpha_s)}(Y_1 - G_2) + \frac{\partial m_1}{\partial G_2} m_1 \right\}$
results of the above described finite period economy, first to an arbitrary $T$ period economy and then letting $T \to \infty$ to an infinite horizon economy. To this end I briefly discuss the proof proposition 3.5 here. I consider a finite $T$ period economy. The two-period environment can be interpreted as results valid for periods $T$ and $T - 1$. By continuing to iterate backwards I can write for any $j \in \{0, 1, 2, ..., T - 1\}$

$$C_{T-j} = (1 - m_{T-j})(Y_{T-j} - G_{T-j+1} - S_{T-j})$$

$$K_{T-j+1} = m_{T-j}(Y_{T-j} - G_{T-j+1} - S_{T-j})$$

where $m_{T-j}$ is an increasing function of $G_{T-j+1}$. Having defined $C_{T-j}$ and $K_{T-j+1}$, given $K_1 > 0$ and $G_1 > 0$, I can define the problem of the incumbent in period $T - j$ as the following$^5$:

$$V^{T-j}(G_{T-j}) = \max_{\{s_{T-j}, G_{T-j+1}, N_j, T-j, \}} U(C_{T-j}) + U(S_{T-j}) + \beta \{\rho V^{T-j+1}(G_{T-j+1}) + (1 - \rho)W^{T-j+1}(G_{T-j+1})\}$$

subject to the expressions for $C_{T-j}$ and $K_{T-j+1}$ defined above. Now taking the first order conditions of the above defined maximization problem and as analogous to the case in the two-period world the first order conditions

$^5$Again I exploit the very special form of the production function of the formal sector here were compared to the general case $\mu = \gamma$. This allows me to write the value functions in terms of the public capital only.
with respect to $S_{T-j}$ and $N_{j,T-j}$ imply $\tau_{n,T-j} = 0$. Moreover, from the first-order condition with respect to $S_{T-j}$ implies

$$U_{S_{T-j}} = (1 - m_{T-j})U_{C_{T-j}}$$

Furthermore, using the form of the utility functions one ends up with $S_{T-j} = \alpha_s(Y_{T-j} - G_{T-j+1})$. As it can be seen from the repetitive pattern of the equations all the results of first period allocations in the two-period model generalize to any period $T-j$. The same is also true for $G_{T-j+1}$ which can be expressed as an increasing function of $\rho$ from the first-order condition with respect to $G_{T-j+1}$. Once $G_{T-j+1}$ is obtained as an increasing function of $\rho$, the rest follows from the above for period 1 in the two-period economy which happens to be the period $T-1$ in a $T$-period economy. Now, using the expressions coming from the first-order conditions and exploiting the fact that all the parameters entering into the formulae for the relevant variables are between 0 and 1, $T \to \infty$, the first period $T-j = 1$ allocations converge to a limit, in which their behaviors with respect to $\rho$ become unchanged.

Proposition 3.6 is an extension of the proposition 3.5 with leisure in the utility function. The only difference between this case and the previous environment is that, even though all the burden of taxation falls on capital again, formal labor is now subsidized. Moreover, as $\rho$ increases, the level of tax subsidy also increases. However, the increase in public investment is
still more than the reduction in the office rent, which increases the capital
tax rate more than the previous case. See Martin (2009) for more details in
an environment with leisure in the utility function.

4.1.3 Computational Algorithm

To compute the interior differentiable Markov-perfect equilibrium, I use\textsuperscript{6}
the global method described in Martin (2009). Given any $\rho$, the basic algo-

rithm is as follows:

1. Define a pair of grids over $K$ and $G$.

2. Guess the decision rules: $K^0, \Gamma^0, \Theta^0_k, \Theta^0_n, N^0_f$.

3. For every $(K,G)$ pair in the grid, solve for $K', G', \tau_k, \tau_n$, and $N_f$, given that $K^0, \Gamma^0, \Theta^0_k, \Theta^0_n, N^0_f$ followed from tomorrow on, using the
equations characterizing Markov-perfect equilibrium. Call the solution

$K^1, \Gamma^1, \Theta^1_k, \Theta^1_n$, and $N^1_f$.

4. Check the convergence of all decision rules. If the convergence error
is not small enough, go back to the previous step and set $K^0 = K^1,

$\Gamma^0 = \Gamma^1, \Theta^0_k = \Theta^1_k, \Theta^0_n = \Theta^1_n$, and $N^0_f = N^1_f$.

Notice that, since I assume differentiability of the policy functions, I
interpolate the points between the grid points to evaluate the policy func-

\textsuperscript{6}I thank Fernando Martin for sharing his codes with me.
tions and calculate the derivatives of them. To be able to do so, I use cubic splines.
4.1.4 Country List

List of Countries Included in the Panel and 80-Country Cross-Section Regressions: Argentina, Australia, Austria, Belgium, Bolivia, Botswana, Brazil, Bulgaria, Canada, Chile, China, Colombia, Costa Rica, Croatia, Czech Republic, Denmark, Dominican Republic, Ecuador, Egypt, El Salvador, Estonia, Finland, France, Germany, Greece, Guatemala, Honduras, Hong Kong, Hungary, India, Indonesia, Ireland, Israel, Italy, Jamaica, Japan, Jordan, Kazakhstan, Kuwait, Latvia, Lebanon, Lithuania, Malaysia, Mexico, Moldova, Morocco, Netherlands, New Zealand, Nicaragua, Norway, Oman, Pakistan, Panama, Paraguay, Peru, Philippines, Poland, Portugal, Romania, Russia, Saudi Arabia, Singapore, Slovakia, Slovenia, South Africa, South Korea, Spain, Sweden, Switzerland, Taiwan, Thailand, Tunisia, Turkey, Ukraine, United Arab Emirates, United Kingdom, USA, Uruguay, Venezuela, Vietnam.

List of Countries Included in the 58-Country Cross-Section Regressions: Argentina, Australia, Austria, Belgium, Bolivia, Brazil, Bulgaria, Canada, Chile, Colombia, Costa Rica, Czech Republic, Denmark, Dominican Republic, Ecuador, El Salvador, Estonia, Finland, France, Germany, Greece, Guatemala, Honduras, Hungary, India, Ireland, Israel, Italy, Jamaica, Japan, Lithuania, Malaysia, Mexico, Moldova, Netherlands, New Zealand, Nicaragua, Norway, Pakistan, Panama, Paraguay, Peru, Philip-
pines, Poland, Portugal, Romania, Russia, Slovakia, Slovenia, South Africa, South Korea, Spain, Sweden, Thailand, Turkey, United Kingdom, USA, Venezuela.

4.2 Appendix B: Appendix to Chapter 2

Proof of Proposition 2.3.2:

First notice that the modern production function is given by

\[ Y_{Mt} = A_{Mt} \eta(S_t) K_M^\theta N_M^{1-\theta}. \]  \hspace{1cm} (4.2.1)

Given \( w \) and \( r_K \) we can write the profit function (for simplicity of notation drop time and modern sector subscripts) as

\[ Y - wN - r_K K. \]  \hspace{1cm} (4.2.2)

The profit per unit is then

\[ 1 - w \frac{N}{Y} - r_K \frac{K}{Y}. \]  \hspace{1cm} (4.2.3)

If we multiply the reciprocal of (33) by \( N \) we obtain

\[ \frac{N}{Y} = \frac{1}{A\eta(S)} \left( \frac{N}{K} \right)^\theta, \]  \hspace{1cm} (4.2.4)
and similarly multiplying the reciprocal of (33) by $K$ we obtain

$$
K/Y = \frac{1}{A\eta(S_t)} \left( \frac{N}{K} \right)^{\theta-1}.
$$

(4.2.5)

Substituting (36) and (37) into (35), we get

$$
1 - \frac{w}{A\eta(S_t)} \left( \frac{N}{K} \right)^{\theta} - \frac{rK}{A\eta(S_t)} \left( \frac{N}{K} \right)^{\theta-1}.
$$

(4.2.6)

Now, maximizing this function with respect to $N$ and $K$, we obtain the following FOCs

$$
- \frac{w}{A\eta(S_t)} K^{-\theta} \theta N^{\theta-1} + \frac{rK}{A\eta(S_t)} K^{1-\theta} (1-\theta) N^{\theta-2} = 0
$$

(4.2.7)

$$
\frac{w}{A\eta(S_t)} K^{-\theta-1} \theta N^{\theta} - \frac{rK}{A\eta(S_t)} K^{-\theta} (1-\theta) N^{\theta-1} = 0.
$$

(4.2.8)

Both of these first order conditions separately imply the same thing which is

$$
\frac{w}{1-\theta} N = \frac{rK}{\theta} K
$$

(4.2.9)

or

$$
\frac{N}{K} = \frac{rK(1-\theta)}{w\theta}.
$$

(4.2.10)

Now what needs to be done is show that
\[ 1 - \frac{w}{A(\eta(S_i))} \left( \frac{N}{K} \right)^\theta \frac{r_K}{A(\eta(S_i))} \left( \frac{N}{K} \right)^{\theta - 1} > 0 \] (4.2.11)

if and only if inequality (18) is satisfied. To prove this, it is enough to show that (18) and (38) are equivalent.

To show this, we take (38) which immediately becomes

\[ 1 > \frac{w}{A(\eta(S_i))} \left( \frac{N}{K} \right)^\theta + \frac{r_K}{A(\eta(S_i))} \left( \frac{N}{K} \right)^{\theta - 1}. \] (4.2.12)

Now using (42) this becomes

\[ 1 > \frac{w}{A(\eta(S_i))} \left( \frac{r_K(1 - \theta)}{w\theta} \right)^\theta + \frac{r_K}{A(\eta(S_i))} \left( \frac{w\theta}{r_K(1 - \theta)} \right)^{1 - \theta} \] (4.2.13)

or

\[ A(\eta(S_i)) > w \left( \frac{r_K(1 - \theta)}{w\theta} \right)^\theta + r_K \left( \frac{w\theta}{r_K(1 - \theta)} \right)^{1 - \theta} \] (4.2.14)

or

\[ A(\eta(S_i)) > w^{1-\theta} r_K^\theta (1 - \theta)^{\theta - \theta} + w^{1-\theta} r_K^\theta (1 - \theta)^{\theta - 1} \theta^{1-\theta} \] (4.2.15)

or

\[ A(\eta(S_i)) > \left( \frac{r_K}{\theta} \right)^\theta \left( \frac{w}{1 - \theta} \right)^{1 - \theta} (1 - \theta + \theta) \] (4.2.16)
which is simply

\[ A > \frac{1}{\eta(S_i)} \left( \frac{rK}{\theta} \right)^{\theta} \left( \frac{w}{1 - \theta} \right)^{1 - \theta} \]  

(4.2.17)

which (with the subscripts) is what we wanted to show.

4.3 Appendix C: Appendix to Chapter 3

Data for GDP, population, investment are taken from the national accounts data of the State Planning Organization which is available at http://www.dpt.gov.tr, and for hours of work data we used the Conference Board and Groningen Growth and Development Centre’s Total Economy Database. The Total Economy Database is available at www.conference-board.org/economics.

The data on consumption of fixed capital which we use to calculate the depreciation to GDP ratio is from national accounts data at www.sourceoecd.org.

For tax exercises in this framework Conesa, Kehoe and Ruhl (2007) describe a very simple procedure to obtain consumption, capital and labor tax series from OECD country tables. Their model is a little different then the methodology suggested by Mendoza, Razin and Tsar (1994).\(^7\) Even tough, Turkey is also an OECD member, revenue statistics for Turkey is far from being complete. Also, even tough there are some studies (such as Gurgel et. al. (2007), and Carey and Tchilinguirian (2000)) estimating capital, labor and consumption taxes for Turkey for one or two specific years,

\(^7\) See the corresponding papers for discussion.
to our knowledge there aren’t any long terms tax series available for Turkey.

To overcome this problem, we do the following:

First, following Conesa, Kehoe and Ruhl (2007), we create a series of $\tau_{Ct}$ by using the following formula

$$\tau_{Ct} = \frac{R_{\text{con},t}}{C_t - R_{\text{con},t}}$$

(4.3.1)

where $R_{\text{con},t}$ is simply the revenue from general taxes on goods and services plus excise taxes which is available at the Turkish Revenue Administration website$^8$ and $C_t$ is simply consumption of households and nonprofit institutions serving households available through national accounts. For the capital and labor taxes, we simply use the generated $\tau_{Ht}$ and $\tau_{Kt}$ series by Cicek and Elgin (2009). Then, we do two analysis with taxes, one taking the average of taxes over the period (1968 - 2004) and running the model with constant taxes. This case is denoted in tables 1 and 2 by tax 1. The second exercise, instead uses the actual tax series that we have generated and is denoted in tables 1 and 2 by tax2. Moreover, for all the tables and figures we take natural logarithm of all the variables and calculate the relevant statistics with these variables.

$^8$www.gib.gov.tr
Bibliography


133


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